APPENDIX A: Mathematical Formulation of MDCEV Models

We provide a brief formulation of the econometric structure of the traditional MDCEV model and then extend the discussion to the formulation for MMDCEV and SMDCEV.

Basic Structure and Traditional MDCEV Model

It is reasonable to expect that the money allocated to different expenditure categories depends on the marginal utility that households derive from spending in those categories. Let us consider that there are *K* different expenditure categories that a household can potentially allocate its money to. If x_k represents the allotted non-negative amount of the total budget to each expenditure category *k* (including savings), the total utility derived from such allocation can be expressed in the following additive non-linear functional form (Bhat, 2008):

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\};\ \psi_k > 0, \alpha_k \leq 1, \gamma_k > 0 \qquad \text{(A.1)}$$

where $U(\boldsymbol{x})$ is a quasi-concave, increasing, and continuously differentiable function with respect to the expenditure quantity (Kx1) - vector $\boldsymbol{x}$ ($x_k \geq 0$ for all *k* alternatives), γ_k and α_k are parameters associated with alternative *k*. ψ_k represents the baseline marginal utility. Through this term, the effect of observed and unobserved alternative attributes, decision-maker attributes, and the choice environment attributes may be introduced as $\psi_k = exp(\beta' z_k + \varepsilon_k)$, where z_k represents the vector of exogenous variables and ε_k captures the idiosyncratic characteristics that affect the baseline utility. γ_k enables corner solutions while simultaneously influencing satiation and α_k influences satiation only. Note that the above utility function is formulated considering absence of outside goods (goods that is always consumed).

If, however, an outside goods is present, the utility function can be modified as follows using the same notational preliminaries:

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1} \exp(\varepsilon_1) \{ (x_1 + \gamma_1)^{\alpha_1} \} + \sum_{k=2}^{K} \frac{\gamma_k}{\alpha_k} exp(\beta' z_k + \varepsilon_k) \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} \qquad \text{(A.2)}$$

In the above formula, we need $\gamma_1 \leq 0$, while $\gamma_k > 0$ for k > 1. Also, we need $(x_1 + \gamma_1) > 0$. The magnitude of γ_1 may be interpreted as the required lower bound (or a "subsistence value") for consumption of the outside goods. In the above baseline parameter expression, the term ε_1 is an idiosyncratic term assumed to be identically and independently standard type I extreme-value distributed across households, as well as independent of the terms in the baseline parameter expression for other alternatives (inside goods).

It is very challenging to identify γ_k and α_k simultaneously in empirical applications for the outside and inside goods (see, Bhat, 2008 and Bhat and Eluru, 2010 for an elaborate discussion on the issue). Usually, the analyst can choose to estimate satiation using either γ_k or α_k, since these two parameters have similar role in terms of allowing for satiation. Depending on the chosen parameter structure for estimation, different utility structures can be estimated and the selection of the most appropriate form is based on statistical considerations.

If only γ_k parameters are estimated the utility simplifies to *γ-profile*

$$U(\boldsymbol{x}) = \exp(\varepsilon_1)\, ln\{(x_1 + \gamma_1)\} + \sum_{k=2}^{K} \gamma_k exp(\beta' z_k + \varepsilon_k)\left\{ln\left(\frac{x_k}{\gamma_k} + 1\right)\right\} \quad \text{(A.3)}$$

Similarly, if only α_k parameter are estimated, the corresponding utility expression collapses to *α-profile*

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1}\exp(\varepsilon_1)\,\{{x_1}^{\alpha_1}\} + \sum_{k=2}^{K} \frac{1}{\alpha_k} exp(\beta' z_k + \varepsilon_k)\{(x_k + 1)^{\alpha_k} - 1\} \quad \text{(A.4)}$$

Let V_k be the alternative utility. The expressions for V_k for *γ-profile* and *α-profile* utility forms are as below:

$$V_k = \beta' z_k - ln\left(\frac{x_k^*}{\gamma_k} + 1\right) - ln\, p_k (k \geq 2);\; V_1 = -ln(x_1 + \gamma_1) \quad \text{(A.5)}$$

$$V_k = \beta' z_k + (\alpha_k - 1)\, ln(x_k^* + 1) - ln\, p_k (k \geq 2);\; V_1 = (\alpha_1 - 1)\, ln(x_1^*) \quad \text{(A.6)}$$

We would assume (following Bhat, 2005 and Bhat, 2008) that ε_k's are independently and identically distributed across alternatives with a scale parameter of σ. Given the values of the alternative utilities for the two profiles, the probability expression for the expenditure allocation to the first *M* of the *K* goods(*M* ≥1) is:

$$P(e_1^*, e_2^*, e_3^*, \dots, e_M^*, 0,0,0, \dots, 0) = \frac{1}{\sigma^{M-1}}\left[\prod_{i=1}^{M} C_i\right]\left[\sum_{i=1}^{M}\frac{1}{C_i}\right]\left[\frac{\prod_{i=1}^{M} exp^{\frac{V_i}{\sigma}}}{\left(\sum_{k=1}^{K} exp^{\frac{V_k}{\sigma}}\right)^M}\right](M-1)! \qquad \text{(A.7)}$$

where, $C_i = \frac{1-\alpha_i}{e_i^* + \gamma_i p_i}$.

In the traditional MDCEV model, the scale parameter σ is set to 1 for normalization.

Scaled MDCEV Model

In our context, due to the inherent differences across the expenditure databases across years and different economic conditions, we can estimate the scale parameter provided we normalize σ for one year. The σ is parameterized as $\exp(\delta y)$ where *y* is the vector of time elapsed variable as well as the annual economic indicators and δ is the corresponding coefficient vector to be estimated. The δ parameters are significant when they are different from 0 as that would imply that the scale parameter will be different from 1. The same expression in Equation A.7 is adopted with the appropriate σ for probability and likelihood computations.

Mixed MDCEV Model

The mixed MDCEV model accommodates unobserved heterogeneity in the effect of exogenous variables (random coefficients structure) and correlations across alternatives (error correlations structure). The baseline parameter expression for the inside alternatives in Equation A.2 can be expressed as follows:

$$\psi_k = exp\{(\beta_k{}' + \alpha_k{}')\, z_k + \eta' w_k + \xi_k\} \qquad \text{(A.8)}$$

In the above equation, β' and α' are column vector of parameters, where β' represents the mean effect and α' represents household level disturbance of the coefficient. The term $\eta' w_k$ constitutes

the mechanism to generate household level correlation across unobserved utility components of the alternatives. In this component, w_k is specified to be a column vector of dimension *H* with each row representing a group *h* (*h=1, 2, ...,* H) of alternatives sharing common household-specific unobserved components and the vector η' may be specified as a *H*-dimensional realization from a multivariate normally distributed random vector *η,* $\eta \sim N(0, \Omega)$. As before, the component, ξ_k is assumed to be independently and identically Gumbel distributed across households. For complete formulation of likelihood for the MMDCEV model see, Bhat and Eluru (2010).

The parameters of MMDCEV model are estimated using maximum simulated likelihood procedure. Specifically, scrambled Halton sequence is used to draw realizations from the population normal distribution. In this research, the stability of the parameter estimates was tested using varying number of Halton draws per observation for the specifications considered, and the results were found to be stable with 100 draws.

Appendix B: Dependent Variable Definition

Table B.1: Dependent Variables

No	Expenditure Category	Definition
1.	Food (FD)	costs incurred from purchase of food and non-alcoholic beverages from grocery stores as well as from restaurants
2.	Shelter (HOU)	rent, regular mortgage payments, condominium charges, property taxes, and home-owners' insurance premiums
3.	Secondary accommodation (SECH)	expenditure for owned vacation home and lodging while away from home (overnight or longer)
4.	Utilities (UTL)	water and sewage charges, electricity, natural gas and other fuel (such as propane and wood for barbeques), telephone, cellular, internet, and postal service costs
5.	Alcohol and tobacco products (ATP)	total expenditure for all tobacco products and smokers' supplies, alcoholic beverages prepared at home as well as purchased and consumed in restaurants and bars
6.	Clothing (CL)	expenditure on purchasing clothes and clothing services (laundry and dry cleaning)
7.	Personal care (PC)	personal care supplies, equipment, and services
8.	Household maintenance and operation (HHMO)	expenses for household furniture and décor wares (such as rugs, curtains), supplies, services, accessories, and household maintenance and operation equipment

9.	Entertainment and recreation (ENT)	cost of home entertainment/sports/hobby equipment and associated services, admission fares to movies and live events, club membership fees, and recreational trip expenses
10.	Education (ED)	costs of books, education supplies, and tuition fees
11.	Health care (HL)	hospital expenses, cost of health care supplies and goods, prescription medicines and pharmaceutical products, eye and dental-care goods and services, health insurance premiums, and other medical services
12.	Business services and welfare activities (BSWA)	total expenditure on financial services, union and professional dues, and charitable contributions
13.	Automobile acquisition (AUTO)	net purchase price paid for automobiles after deducting any trade-in allowance or separate sales and costs for renting and leasing vehicles
14.	Recreational vehicle (RECV)	purchase/rent and operation of recreational vehicles
15.	Gasoline costs (GAS)	gasoline and other fuel expenses for owned and leased vehicles
16.	Vehicle insurance costs (VEHI)	total public and private vehicle insurance premiums paid for owned and leased automobiles
17.	Vehicle operation and maintenance (VOP)	expenses accrued from maintenance and repair operations, garage rent and parking fee, and purchase of accessories
18.	Public transportation (PT)	local and commuter transportation costs
19.	Non-motorized transport (NMT)	purchase cost of bikes, parts and accessories as well as maintenance and repair costs
20.	Intercity travel (INTT)	fare of airplane, train and highway bus travel
21.	Savings	created by subtracting the total annual expenditure from the total gross income. If savings were negative, then the savings variable was coded as zero