

CHOICE MODELS IN TRANSPORTATION

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ABSTRACT

Decision makers (DM) – individuals, households, and firms among others - make several decisions as part of their life cycle. In understanding these decision processes that are usually discrete outcome variables, researchers across various fields including economics, psychology and transportation have developed different frameworks, referred to as Choice models. In this chapter, we illustrate how choice or outcome models can provide data supported analysis mechanisms in transportation. Specifically, we present models from three frameworks (a) random utility maximization and (b) random regret minimization approach and (c) ordered response structure. The study provides a brief overview of multiple case studies from different transportation contexts. We also discuss the application of choice models for travel mode share analysis.

Keywords: Random utility approach, multinomial logit model, mixed logit, random regret minimization

1 INTRODUCTION

Decision makers (DM) – individuals, households, and firms among others - make several decisions as part of their life cycle. These decisions could be short time decisions such as *what mode of travel to consider to arrive at the next activity location* (for individuals) or long-term decisions such as *what house to purchase* (for a household) or *where to establish a place of residence or commercial enterprise* (for a household or a firm respectively). In understanding these decision processes that are usually discrete outcome variables, researchers across various fields including economics, psychology and transportation have developed different frameworks, referred to as Choice models. These frameworks relying on assumptions about the choice environment and the decision maker provide a mathematical representation to study choice behavior. The formulation of the choice model will vary based on the assumptions incorporated within the representation structure. These mathematical formulations are employed to analyze data from wide-ranging fields including economics, statistics, marketing, transportation, psephology, political science, and biostatistics.

In the transportation field, a large number of diverse examples of choice behavior is often encountered. At the individual level, the various choices situations encountered include: (a) travel mode choice - an individual chooses a transportation mode from a set of available transportation modes (such as car, bus, and walk), (b) route choice – a road user chooses a route from multiple alternatives for a trip (such as different Google Maps generated routes), (c) activity choice - an individual chooses an activity from a series of activities for the day (such as leisure and shopping), and (d) freight shipment size – an individual shipper determines the size of their shipment (such as under 50 pounds, 50-200 pounds and greater than 200 pounds). At the household level a sample of choices considered include (a) residential location choice – a household chooses a residential unit to buy or rent (spatial aggregated alternatives from the urban region such as traffic analysis zones or parcels), and (b) vehicle type choice – a household chooses to buy a vehicle defined by a combination of type (such as sedan minivan or coupe), make (such as Chevy, or Honda) and fuel type (electric or gasoline). At the firm level sample of choices evaluated include: (a) firm size and structure – a firm selects a size defined by the number of employees and hierarchy structure, and (b) firm location choice – a firm determines a location based on various incentives or tax implications. The examples illustrate the range and complexity of these choices.

In this chapter, we illustrate how choice or outcome models provide data supported analysis mechanisms for such choice contexts. The rest of the chapter is organized as follows: Section 2 summarizes mathematical frameworks commonly used for choice analysis in transportation. In Section 3, we provide a brief overview of case studies examining different choice contexts in transportation, Section 4 provides details of choice model application for travel mode share applications. Finally, section 5 concludes the chapter.

2 CHOICE MODEL ARCHITECTURE AND MATHEMATICAL FORMULATIONS

Across the various choice contexts described earlier, DMs have to process the available information regarding the choice environment in arriving at their preferred choice. A DM's choice is dependent on how the DM defines the decision at hand, processes the alternatives (and their attributes) and the decision rule applied to arrive at the choice. The choice problem definition determines the potential alternatives of the choice problem. For example, if the DM's choice context is to determine the travel mode for an out of home activity, the alternatives will possibly include all transportation modes (universal choice set of transportation modes) available in the study region. Of these alternatives, based on the DM's characteristics only a subset are feasible

(feasible choice set). However, based on the attributes of the various alternatives it is possible that the decision maker will only consider a subset of alternatives from the feasible choice set (evoked choice set). The DM evaluates the alternatives in the evoked choice set based on the alternative attributes. For example, in travel mode choice context, alternative attributes include travel time and travel cost.

In choice contexts with smaller number of alternatives in the evoked choice set, DM can compare across alternatives using various decision rules to arrive at the final choice. In other cases, decision makers may consider a small subset of the alternatives to determine their choice. Earlier choice frameworks attempted a process of elimination (using satisfaction criteria Murtaugh and Gladwin, 1980) or ranking approach using lexicographic criteria (Foerster, 1979) to determine the DM's choice. However, these approaches might provide non-unique choices and/or choice outcomes that are impractical. Toward addressing these concerns, a scalar alternative utility based approach was proposed. In this approach, each alternative is accorded a score based on a utility function (usually a linear additive function) of various alternative attributes. The alternative with the highest utility is considered to be the chosen alternative. Through the utility function, the approach allows for trade-offs (compensatory effects) between various attributes. For example, in a utility approach for travel mode choice, increase in travel cost can be compensated by improvements in travel time. While DM's are completely aware of the various attributes considered, analysts only observe the choice process partially. Hence, to allow for the influence of missing or unobserved information, utility is partitioned into two parts – observed component and unobserved component. The observed component accommodates for the impact of variables compiled in the data collection. The unobserved component takes the form of a stochastic error term to represent missing information. The alternative with the largest utility – computed as the sum of the two components - is considered the chosen alternative. Given the inherent stochasticity, the approach is referred to as the Random Utility Maximization (RUM) approach.

RUM choice models represent the most commonly employed frameworks for choice model development. The approach allows for the consideration of trade-offs across various attributes affecting the choice process. This implicit compensatory nature of the formulation allows for a poor performance on an attribute to be compensated by a positive performance on another attribute (Chorus et al., 2008). Several researchers, motivated by research in behavioral economics, have considered alternative decision rules for developing discrete choice models such as relative advantage maximization (Leong and Hensher, 2015), contextual concavity (Kivetz et al., 2004), fully-compensatory decision making (Arentze and Timmermans, 2007; Swait, 2001), prospect theory (PT) (Kahneman and Tversky, 2013; Tversky and Kahneman, 1992), and random regret minimization (RRM) (Chorus, 2010; Chorus et al., 2008). Of these approaches RRM offers a framework that parallels RUM based models and is emerging as alternative modeling framework.

The reader would note that while all the choice examples presented in the introduction are discrete/categorical variables, some of them are potentially ordered discrete variables (for example number of employees)¹. The RUM and RRM approaches are applicable for analyzing all discrete variables. However, for modeling ordered dependent variables, another class of models referred to as ordered response frameworks also emerge as a potential model structure. In the following

¹ Other examples of ordered discrete variables in the field of transportation include: (1) driver and passenger injury severity in traffic collisions, (2) household vehicle (automobile and bicycle) ownership, and (3) activity participation indicators (such as number of tours, number of stops, activity episode participation frequency and activity duration).

discussion, we present the mathematical details of the widely adopted frameworks organized by dependent variable characterization: (a) unordered dependent variable models and (b) ordered dependent variable models.

2.1 Unordered dependent variable models

The unordered model frameworks are applicable for modeling all discrete choice or outcome dependent variables. In this section, we describe the RUM based multinomial logit model and its extensions (mixed logit and latent segmentation based multinomial logit model) followed by a discussion of RRM based multinomial logit model.

2.1.1 Multinomial logit model

Consider that a DM n has to choose an alternative i from a set of alternatives ($j = 1, 2, \dots, J$). With these notation, the utility in the RUM framework is defined as:

$$v_{ni} = \beta_i x_{ni} + \varepsilon_{ni} \quad (1)$$

where v_{ni} is the utility obtained by individual n by selecting alternative i from the choice set j . x_{ni} is the vector of alternative and DM's characteristics, β is a corresponding vector of parameters (including a constant) and ε_{ni} is the stochastic term that captures the unobserved part of the utility. The DM is expected to select an alternative that provides the highest utility. Given the inherent stochasticity involved, the choice of an alternative can only be arrived as the probability that a particular alternative (i in our case) provides the highest utility. The mathematical formula for an alternative probability of choice is affected by the assumptions about the stochastic term (ε_{ni}). Assuming ε_{ni} , to be independent and identically Type I Standard Extreme Value distributed across the dataset simplifies the probability evaluation to the following multinomial logit form

$$P_{ni} = \frac{e^{v_{ni}}}{\sum_{\forall j} e^{v_{nj}}} \quad (2)$$

Under different stochastic assumptions, such as ε_{ni} following a multivariate normal distribution would give rise to multinomial probit model.

The estimation of the parameters (β) is achieved by maximizing the likelihood function across the dataset i.e. maximizing the probability of the chosen alternative across all records. The likelihood function for individual n is defined as

$$L_n = \sum_{\forall j} P_{nj}^{y_{nj}} \quad (3)$$

where P_{nj} is the probability of the individual n choosing j and y_{nj} is an indicator variable that takes the value 1 for chosen alternative, 0 otherwise.

The likelihood function for the entire data is computed as the product of likelihood over all the responses in the data as follows

$$L = \prod_{n=1}^N \sum_{\forall j} P_{nj}^{y_{nj}} \quad (4)$$

For mathematical convenience, the natural logarithm of the likelihood function – log-likelihood - is maximized. The log-likelihood function takes the following form

$$LL = \sum_{n=1}^N \sum_{\forall j} y_{nj} P_{nj} \quad (5)$$

The maximization of the function from Equation 5 is achieved using different non-linear optimization methods. With the increased access to computational resources, several proprietary and open-source software are available for estimating the models such as SPSS, SAS, ALOGIT and R. The reader would note that the likelihood approach and software are similar for all models presented in the remaining discussion.

2.1.2 Mixed logit model

In the multinomial logit model, the vector of parameters estimated (β) is restricted to be the same across all DM's. The restriction is often referred to as the population homogeneity assumption. The mixed multinomial logit model accommodates for heterogeneity effects across DM's by allowing for the vector of parameters to follow a distribution across the dataset (as opposed to a fixed value). Based on the distributional assumption, the parameters that define the distribution are estimated.

In mixed logit models, the random utility formulation takes the following form (building on the formulation from Equation 1):

$$v_{ni} = (\beta_i + \eta_{ni})x_{ni} + \varepsilon_{ni} \quad (6)$$

where η_n is another column vector representing unobserved factors, usually considered to be independent realizations from a normal population distribution ($\eta_n \sim N(0, \sigma^2)$). Then, the probability that any DM will select alternative i for a given value of η can be expressed as:

$$P_{ni}|\eta = \frac{e^{[(\beta + \eta_{ni})x_{ni}]}}{\sum_{\forall j} e^{[(\beta + \eta_{ni})x_{ni}]}} \quad (7)$$

The unconditional probability then can be written as:

$$P_{ni} = \int_{\eta} (P_{ni}|\eta_{ni}) dF(\eta_{ni}|\sigma) \quad (8)$$

where F is the multivariate cumulative normal distribution and σ is a vector of parameters. The integral in the probability expression cannot be evaluated using an analytical approach. Hence, the mixed logit model estimation is approximated using simulation methods for integral evaluations.

The most commonly used method of simulation are Pseudo-Monte Carlo (PMC) simulation and Quasi-Monte Carlo (QMC) simulation (Bhat, 2003). The wide adoption of QMC approaches have resulted in ubiquitous adoption of mixed logit models for analysis in transportation and several other fields.

2.1.3 Latent class multinomial logit model

In the mixed logit model system, while the mean of the random coefficients can be allowed to vary across DM's based on observed variables, the approach usually restricts the variance and the distributional form of a random coefficient to be the same across all drivers. An alternative approach to relax the population heterogeneity assumption is the latent (or sometimes also referred to as endogenous) segmentation approach. In this approach, the DM's are allocated probabilistically to different segments, and segment-specific multinomial models are estimated. Such a segmentation approach is appealing in many respects: (a) each segment is allowed to be identified with a multivariate set of exogenous variables, (b) the probabilistic assignment of DM's to segments explicitly acknowledges the role played by unobserved factors in moderating the impact of observed exogenous variables, and (c) there is no need to specify a distributional assumption for the coefficients (Greene and Hensher, 2003; Yasmin et al., 2014).

Let us consider S homogenous segments of DM's (the optimal number of S is to be determined). We need to determine how to assign the DM's probabilistically to the segments for the segmentation model. The utility for assigning a DM n ($1,2,..N$) to segment s follows the multinomial logit structure as:

$$U_{ns} = \gamma_s z_n + \xi_{ns} \quad (9)$$

where z_n is a matrix of attributes that influences the propensity of belonging to segment s , γ_s is a vector of coefficients and ξ_{ns} is an idiosyncratic random error term assumed to be identically and independently Type 1 Extreme Value distributed across DM's n and segment s . Then the probability that DM n belongs to segment s is given as:

$$P_{ns} = \frac{\exp(\gamma_s z_n)}{\sum_s \exp(\gamma_s z_n)} \quad (10)$$

Within the latent segmentation approach, the probability of DM n choosing alternative i is given as:

$$P_{ni} = \sum_{s=1}^S (P_n(i) | s)(P_{ns}) \quad (11)$$

where $P_n(i)$ represents the multinomial logit probability for selecting alternative i within segment s following notation from Equation 2.

2.1.4 Regret based multinomial logit model

The prevalent framework for developing discrete choice models is the RUM approach. RUM based approaches assume that decision makers prefer routes that provide the highest utility or satisfaction

(Ben-Akiva et al., 1985; McFadden, 1974; Train, 2009). Among these approaches, the regret minimization approach offers a viable alternative approach due to its mathematical simplicity within a semi-compensatory decision framework.

The random regret associated with the choice of alternative i among j alternatives, each characterized by m attributes by the DM n is given as

$$RR_{ni} = \sum_{j \neq i} \sum_{\forall m} \ln \{1 + \exp[\tau_m (x_{njm} - x_{nim})]\} \quad (12)$$

where τ_m denotes the estimable parameter associated with attribute x_m . x_{njm} and x_{nim} denote the values associated with attribute x_m for chosen alternative i and considered alternative j for DM n . In random regret models, the error term is assumed to be identically and independently Type 1 Extreme Value distributed across the dataset, which yields a closed form of probability expression similar to utility based multinomial logit model.

$$P_{ni} = \frac{e^{-RR_{ni}}}{\sum_{\forall j} e^{-RR_{nj}}} \quad (13)$$

The model estimation follows a similar procedure of maximizing likelihood described earlier. Finally, the reader would note that similar extensions described in Section 2.1.2 and 2.1.3 can be accommodated for RRM frameworks to relax population homogeneity assumptions.

2.2 Ordered dependent variable models

The ordered response models represent the decision process under consideration using a single latent propensity. The choice probabilities are determined by partitioning the uni-dimensional propensity into as many categories as the dependent variable alternatives through a set of thresholds. The reader would note that the distributional assumptions of the unobserved component of the latent propensity determines the exact formulation of the model. The prevalent mechanism to analyze ordered discrete variables including ordered logit and generalized ordered logit models are presented in this section.

2.2.1 Ordered logit model

Let n ($n = 1, 2, \dots, N$) and j ($j = 1, 2, \dots, J$) be the indices to represent decision makers and alternatives, respectively. In the traditional OL model, alternative levels (y_n) are assumed to be associated with an underlying continuous latent variable (y_n^*). This latent variable is typically specified as the following linear function:

$$y_n^* = \alpha z_n + \epsilon_n, \quad y_n = i, \quad \text{if } \tau_{i-1} < y_n^* < \tau_i \quad (14)$$

where, y_n^* is the latent propensity for DM n choosing an alternative level i , x_n is a vector of exogenous variables, α is a vector of coefficients to be estimated and ϵ_n is a random disturbance term assumed to be standard logistic. The latent propensity y_n^* is mapped to the observed ownership levels y_n by τ thresholds ($\tau_0 = -\infty, \tau_J = +\infty$) with the following ordering conditions:

$(-\infty < \tau_1 < \tau_2 < \dots < \tau_{j-1} < +\infty)$. Given these relationships across the different parameters, the resulting probability expression takes the following form:

$$P_{ni}(y_n = j) = \Lambda(\tau_n - \alpha x_n) - \Lambda(\tau_{j-1} - \alpha x_n) \quad (15)$$

where, $\Lambda(\cdot)$ is the standard logistic cumulative distribution function (see (Greene and Hensher, 2010; Train, 2009) for more details). A standard normal distributional assumption for ϵ_n would result in an ordered probit model system.

2.2.2 Generalized ordered logit model

The generalized ordered response model relaxes the constant threshold across population restriction to provide a flexible form of the traditional OL model. The basic idea of the GOL is to represent the threshold parameters as a linear function of exogenous variables (Eluru et al., 2008; Maddala, 1983; Srinivasan, 2002; Terza, 1985). Thus, the thresholds are expressed as:

$$\tau_{n,j} = \text{function of } (Z_{nj}) \quad (16)$$

where, Z_{nj} is a set of exogenous variable (including a constant) associated with j th threshold. Further, to ensure the accepted ordering of observed discrete severity $(-\infty < \tau_{i,1} < \tau_{i,2} < \dots < \tau_{i,j-1} < +\infty)$. We employ the parametric form employed by (Eluru et al., 2008):

$$\tau_{n,j} = \tau_{n,j-1} + \exp(\delta_{nj} Z_{nj}) \quad (17)$$

where δ_{nj} is a vector of parameters to be estimated. The remaining structure and probability expressions are similar to the OL model. For identification reasons, we need to either suppress the latent propensity of one of the δ_{nj} vectors. The model estimation follows a similar procedure of maximizing likelihood described earlier. Also, the reader would note that similar extensions described in Section 2.1.2 and 2.1.3 can be accommodated for ordered response frameworks to relax population homogeneity assumptions.

3 CASE STUDIES

To illustrate the diverse application of choice models in transportation, we present a brief summary of choice model application for different transportation empirical contexts². Table 1 provides the summary of studies with information on study region, transportation topic of interest, data elicitation approach employed in the study, dependent variable used along with the details of the alternatives considered, choice model estimated, and the independent variables found to affect the choice process. Several observations can be made from the summary presented in Table 1. First, choice models are applied for examining different transportation dimensions such as passenger and freight travel model choice, route choice, activity type choice, residential location choice, electric and autonomous vehicle purchase decisions, activity destination choice and comparison of

² The reader would note that an exhaustive review of studies employing choice models is beyond the scope of the chapter.

travel times by mode. Second, the dependent variables vary considerably in terms of the number of alternatives ranging from 2 through 30. Third, the methodologies considered in these studies span the spectrum of choice models including simple binary logit models to panel mixed multinomial logit models. The studies also encompass RUM and RRM based models. Finally, a detailed summary of the independent variables considered for each study is included in the table. This summary on factors highlight how the adoption of these choice model frameworks can offer insights on a host of independent variables such as (a) DM's socio-economic and socio-demographics, (b) alternative specific characteristics such as mode characteristics, and dwelling unit characteristics and (c) choice environment attributes such as transportation network infrastructure attributes, built environment factors, and regional and environmental factors.

Table 1 Literature Review of Choice Models in Transportation

Study	City / Country	Field of Study	Data Source	Dependent Variable	Modeling approach	Factors affecting the choice
Puan et al., 2019	Johor Bahru (Malaysia)	Mode choice	Survey	Travel Mode (car or bus)	Binary Logit	Demographic and socio-economic characteristics: age, gender, education, employment, income, household size, and vehicle ownership, Mode characteristics: travel time, travel cost, toll cost, parking cost, parking availability, number of transfers, comfort, reliability, overall quality of bus service, and bus coverage
Zhao et al., 2019	China	Route choice	Survey	Driver's route choice response to incident information through VMS	Multinomial logit model	Demographic and socio-economic characteristics: gender, driving experience, education, occupation, income Trip characteristics: trip purpose, Road information: alternate route information (delay, congestion message by text) VMS characteristics: color, text, graph
Shabanpour et al., 2017	Chicago, USA	Activity type	URACS Survey	Activity start times with 6 time periods of the day	Hybrid RUM-RRM model	Travel time, age, gender, income, employment, travel mode, activity location, and activity duration.
Keya et al., 2018	USA	Freight mode choice	CFS	Shipping mode (hire truck, private truck, air, courier and others)	Hybrid Regret-Utility based MNL, Latent Class model	Mode Characteristics: Shipping cost and shipping time Freight Characteristics: Type of shipment and value of shipment Transportation Network and Demographic variables: Type of region, climate, road network density, population density, employment density, poverty level and seaports at origin and destination location.
Marois et al., 2019	Montreal (Canada)	Residential location	National Household Survey data	Residential location choice (from 30 alternatives)	Mixed Logit	Demographic and socio-economic characteristics: Budget, previous residence, housing cost, income, and family structure. Dwelling characteristics: Type of the unit, number of rooms, state of repair and age of the house Neighborhood characteristics: Urban morphology and amenities.
Nickkar et al., 2019	Maryland, USA	Electrical vehicle choice	Survey	Ownership of electrical vehicles by	Multinomial Logit	Demographic and socio-economic characteristics: Age, education, income, marital status, race, political affiliation, household size, number of vehicles in household.

Study	City / Country	Field of Study	Data Source	Dependent Variable	Modeling approach	Factors affecting the choice
				female drivers		
Jiang et al., 2019	Japan	Autonomous vehicle	Survey	Ownership of autonomous vehicles	Mixed Logit	Alternative characteristics: Additional purchase cost, Insurance reduction rate, parking cost reduction, Choice environment attributes: release time of AVs to market, penetration rate of AVs, driving experience (short and long terms), age and income of the respondent.
Hasnat et al., 2019	Central Florida Region, USA	Destination choice	Location based social media data	Destination choice among a choice set of 30 census tracts	Panel Latent Segmentation Multinomial Logit	Origin and destination characteristics: residential, industrial, recreational, office, agricultural, land use mix, per capita income, number of schools, hospitals and civic centers. Distance between origin and destination
Faghih-Imani et al., 2017	New York, USA	Bike share	New York City bike data	Faster mode of travel (bicycle or cab)	Panel Mixed Multinomial Logit	Trip characteristics: Time of day, trip distance, whether the trip includes crossing a bridge or not Origin and destination attributes: Station capacity, length of cycling facilities, length of streets, number of restaurants, population density, employment density, presence of transit station.

4 MODE SHARE MODELS

In the transportation field, travel mode share models represent the most common application of choice models. Among choice models, the most commonly employed frameworks include unordered model systems such as RUM based multinomial logit and RUM based Mixed Logit. Recently RRM based multinomial logit and mixed logit have also been applied. These model systems provide an understanding of the impact of various attributes affecting mode choice.

Traditionally, travel mode choice models are estimated using household travel survey data. In these surveys, individual travel diaries compile information on travel mode choice for every out of home travel episode. These travel episodes can be modeled individually as a trip or can be considered as a chain of trips in the form of trip chain or tour. Depending on the modal aggregation by trip or tour, the model developed will either represent a trip level mode choice or tour level mode choice. In all these models, the data compiled from the survey provides only a subset of information necessary for model estimation. To elaborate, the level of service information (such as travel time and cost by mode) is usually provided by respondents only for the chosen alternative. In the model estimation process, the analyst will need to build the other alternatives under consideration for the specific record (trip or tour) and then generate level of service measures for all these alternatives. The generation of level of service measures is likely to be challenging. For instance, the travel cost of a trip by public transit is not likely to be easy to estimate if the individual uses a monthly or annual pass. The analyst will need to estimate the number of potential instances the individual will use transit for in determining the record level cost. Similar challenges exist in evaluating automobile record level travel costs as maintenance and operational costs vary substantially based on the vehicle used for travel as well as the frequency of use. Thus, in transportation, analysts typically use urban regional traffic analysis zone (TAZ) level travel time and travel cost matrices for analysis. These matrices are generated using traffic assignment results from regional travel demand models. To be sure, the approach does result in some conceptual challenges. For example, changes to travel mode preferences will modify the level of service matrices and vice versa. Hence, it is prudent to consider multiple iterations between mode choice and the traffic assignment process to ensure the level of service estimates used in mode choice are realistic.

The model estimation will consider several variables including level of service attributes, DM socio-demographic and socio-economic attributes, transportation infrastructure and built environment variables. The specification of mode choice models for level of service attributes is different from other discrete choice contexts. Usually, the coefficients for independent variables are estimated specific to each alternative. However, for level of service attributes, it is useful to consider a generic parameter across all alternatives. For example, the generic parameter ensures that level of service parameters for variables such as travel time and travel cost are same across all modes. Further, the presence of generic parameters allows us to evaluate the trade-off between various mode specific level of service attributes. The trade-off between travel time and cost, defined as the value of time (VOT) or willingness to pay (WTP) is generated using mode choice estimates. For RUM based multinomial model system, the measure is defined as

$$VOT_{RUM} = \frac{\beta_{tt}}{\beta_{tc}} \quad (18)$$

where β_{tt} and β_{tc} are estimates of travel time and travel cost respectively from the RUM based multinomial logit model.

For RRM framework, the trade-offs are dependent on levels of attributes and generated as:

$$VOT_{RRM} = \frac{\sum_{j \neq i} -\beta_{tt} \left(1 + \frac{1}{\exp[\beta_{tt}(t_j - t_i)]} \right)}{\sum_{j \neq i} -\beta_{tc} \left(1 + \frac{1}{\exp[\beta_{tc}(c_j - c_i)]} \right)} \quad (19)$$

where β_{tt} and β_{tc} are estimates of travel time and travel cost respectively from the RRM based multinomial logit model, t_i and t_j represent the travel time attributes for the chosen route i and considered route j , respectively. c_i and c_j are represent the travel time attribute for the chosen route i and considered route j , respectively. To be sure, these value of time expressions will need to be appropriately modified for mixed logit and latent segmentation models.

5 CONCLUSION

The current chapter provides an overview of model frameworks employed to model Decision Maker choice behavior in the context of transportation. Given the wide range of choice outcomes examined in transportation, multiple choice models useful for analyzing unordered and ordered discrete variables are presented. The unordered frameworks described include random utility based multinomial logit model and random regret based multinomial logit model, and their mixed and latent segmentation variants. For ordered discrete variables, we present the traditional ordered model and its generalized variant. A brief overview of case studies covering a diverse set of choice contexts is also presented. The description of choice models was restricted due to space constraints. Several advanced modeling approaches (such as panel models, generalized extreme value extensions of traditional models) that build on the frameworks described in the chapter are developed in recent years in economics and transportation fields.

6 REFERENCES

- Arentze, T. and Timmermans, H. 2007. Parametric action decision trees: Incorporating continuous attribute variables into rule-based models of discrete choice. *Transportation Research Part B: Methodological* 41(7), 772-783.
- Ben-Akiva, M.E., Lerman, S.R. and Lerman, S.R. (1985) *Discrete choice analysis: theory and application to travel demand*, MIT press.
- Bhat, C.R. 2003. Simulation estimation of mixed discrete choice models using randomized and scrambled Halton sequences. *Transportation Research Part B: Methodological* 37(9), 837-855.
- Chorus, C.G. 2010. A new model of random regret minimization. *European Journal of Transport and Infrastructure Research* 10(2).
- Chorus, C.G., Arentze, T.A. and Timmermans, H.J. 2008. A random regret-minimization model of travel choice. *Transportation Research Part B: Methodological* 42(1), 1-18.
- Eluru, N., Bhat, C.R. and Hensher, D.A. 2008. A mixed generalized ordered response model for examining pedestrian and bicyclist injury severity level in traffic crashes. *Accident Analysis & Prevention* 40(3), 1033-1054.
- Faghih-Imani, A., Anowar, S., Miller, E.J. and Eluru, N. 2017. Hail a cab or ride a bike? A travel time comparison of taxi and bicycle-sharing systems in New York City. *Transportation Research Part A: Policy and Practice* 101, 11-21.

- Foerster, J.F. 1979. Mode choice decision process models: a comparison of compensatory and non-compensatory structures. *Transportation Research Part A: General* 13(1), 17-28.
- Greene, W.H. and Hensher, D.A. 2003. A latent class model for discrete choice analysis: contrasts with mixed logit. *Transportation Research Part B: Methodological* 37(8), 681-698.
- Greene, W.H. and Hensher, D.A. (2010) *Modeling ordered choices: A primer*, Cambridge University Press.
- Hasnat, M.M., Faghih-Imani, A., Eluru, N. and Hasan, S. 2019. Destination choice modeling using location-based social media data. *Journal of choice modelling* 31, 22-34.
- Jiang, Y., Zhang, J., Wang, Y. and Wang, W. 2019. Capturing ownership behavior of autonomous vehicles in Japan based on a stated preference survey and a mixed logit model with repeated choices. *International Journal of Sustainable Transportation* 13(10), 788-801.
- Kahneman, D. and Tversky, A. (2013) *Handbook of the fundamentals of financial decision making: Part I*, pp. 99-127, World Scientific.
- Keya, N., Anowar, S. and Eluru, N. 2018. Freight mode choice: A regret minimization and utility maximization based hybrid model. *Transportation Research Record* 2672(9), 107-119.
- Kivetz, R., Netzer, O. and Srinivasan, V. 2004. Alternative models for capturing the compromise effect. *Journal of marketing research* 41(3), 237-257.
- Leong, W. and Hensher, D.A. 2015. Contrasts of relative advantage maximisation with random utility maximisation and regret minimisation. *Journal of Transport Economics and Policy (JTEP)* 49(1), 167-186.
- Maddala, G. 1983. Methods of estimation for models of markets with bounded price variation. *International Economic Review*, 361-378.
- Marois, G., Lord, S. and Morency, C. 2019. A mixed logit model analysis of residential choices of the young-elderly in the Montreal metropolitan area. *Journal of Housing Economics* 44, 141-149.
- McFadden, D. 1974. The measurement of urban travel demand. *Journal of public economics* 3(4), 303-328.
- Murtaugh, M. and Gladwin, H. 1980. A hierarchical decision-process model for forecasting automobile type-choice. *Transportation Research Part A: General* 14(5-6), 337-347.
- Nickkar, A., Shin, H.-S. and Farkas, A. 2019. Analysis of Ownership and Travel Behavior of Women Who Drive Electric Vehicles: The case of Maryland. arXiv preprint arXiv:1911.03943.
- Puan, O., Hassan, Y., Mashros, N., Idham, M., Hassan, N., Warid, M. and Hainin, M. 2019. *Transportation mode choice binary logit model: a case study for Johor Bahru city*, p. 012066, IOP Publishing.
- Shabanpour, R., Golshani, N., Auld, J. and Mohammadian, A. 2017. Dynamics of activity time-of-day choice. *Transportation Research Record* 2665(1), 51-59.
- Srinivasan, K.K. 2002. Injury severity analysis with variable and correlated thresholds: ordered mixed logit formulation. *Transportation Research Record* 1784(1), 132-141.
- Swait, J. 2001. A non-compensatory choice model incorporating attribute cutoffs. *Transportation Research Part B: Methodological* 35(10), 903-928.
- Terza, J.V. 1985. A Tobit-type estimator for the censored Poisson regression model. *Economics Letters* 18(4), 361-365.
- Train, K.E. (2009) *Discrete choice methods with simulation*, Cambridge university press.
- Tversky, A. and Kahneman, D. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty* 5(4), 297-323.

- Yasmin, S., Eluru, N., Bhat, C.R. and Tay, R. 2014. A latent segmentation based generalized ordered logit model to examine factors influencing driver injury severity. *Analytic methods in accident research* 1, 23-38.
- Zhao, W., Quddus, M., Huang, H., Lee, J. and Ma, Z. 2019. Analyzing drivers' preferences and choices for the content and format of variable message signs (VMS). *Transportation research part C: emerging technologies* 100, 1-14.