Joint Modeling of Traffic Incident Duration Components (Reporting, Response, and Clearance Time): A Copula Based Approach

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ABSTRACT

The current study develops a tri-variate framework that accommodates for inherent dependencies across the various components of incident duration. A unique ordered response model structure (sometimes referred to as grouped ordered response model) is introduced for modeling three durations - reporting time, response time, and clearance time - in our study. Further, as opposed to employing a simulation oriented multivariate model approach, we propose and estimate a copula based methodology that allows for a closed-form probability computation. The approach is the first application of this model for incident duration analysis. The proposed copula framework is estimated to identify factors affecting incident duration components from a host of characteristics including incident characteristics, traffic conditions, roadway, and environmental characteristics. The data for the analysis is obtained from the 2015 Florida Department of Transportation incident database. The model estimates were also augmented by conducting policy analysis by generating 3-dimensional representation of incident frequencies as a function of reporting, response, and clearance time.

INTRODUCTION

Traffic congestion can generally be attributed to either recurring or non-recurring events. Congestion arising from recurring events is generally a result of mismatched transportation demand and supply (or capacity). Non-recurring congestion, on the other hand, is a result of unexpected (or irregular) events such as abandoned vehicles, adverse weather, spilled loads, highway debris, and traffic crashes. The potential solutions for congestion arising from these two sources are vastly different. In our research, we focus our attention on non-recurrent congestion. Non-recurring congestion is responsible for approximately one-quarter of all traffic delay on US roadways (*1*). According to Roper (*2*), every minute that a freeway travel lane is blocked results in 4 to 5 minutes of traffic delay after the incident is cleared. The U.S. Department of Transportation Strategic Plan for Fiscal-Year 2010-2015 reports that 2.8 billion gallons of gasoline is consumed every year in US due to incident-related congestion events (*3*). Moreover, longer incident durations can increase the risk of secondary incidents (*4*). Consequently, transportation agencies are developing traffic incident management strategies to reduce the overall duration of incidents to minimize their impacts on travelers and environment.

The overall incident duration, as identified by the Highway Capacity Manual (*5*), is composed of the following four phases: Notification time, Response time, Clearance time and Traffic recovery time (*6*). Incident clearance (third phase) is usually the longest component of the incident duration time (*4*). The traffic recovery time (fourth phase) is a function of total duration of the first three phases and the traffic demand on the facility. Given the importance of different phases of incident duration, the objective of the proposed research effort is to study the factors influencing incident reporting, response and clearance times – with a goal of reducing the congestion impacts of non-recurring events while providing improved traffic incident management plans. An important factor affecting incident clearance is the personnel involved. In the state of Florida, in addition to the traditional agencies, a road ranger service patrol assists in the incident clearance process. Specifically, Florida Department of Transportation (FDOT) offers a unique service via road ranger service patrol to offer free assistance to road users on highways to reduce delay while enhancing safety for the public. Since its inception, the road ranger service has offered nearly 4.3 million assists (*7*). The objectives of the program include reducing traffic crashes, assisting the Florida Highway Patrol to reduce incident duration, providing assistance to disabled or stranded vehicles, removing road debris, and increasing safety at incident sites. Toward meeting these goals, the Road Ranger trucks monitor congested areas and high incident locations of the urban expressway for road debris, traffic crashes or incidents, and stranded vehicles. In this research, we examine incident clearance duration with a specific emphasis on the impact of road ranger service patrol program. The analysis would allow us to make recommendations on the performance of the road ranger patrol while offering recommendations for similar programs in other states.

EARLIER RESEARCH

Several research efforts have examined incident duration as a function of incident characteristics, traffic conditions, and roadway characteristics. Literature in incident management focusing on incident duration is guided by two objectives. The first objective is an emphasis on incident duration prediction. The second objective is to identify the exogenous variables that affect incident duration (*8*). A detailed review of earlier research on incident duration models is summarized in Table 1. The information presented in the table includes the study, study region and data source, outcome variable, type of model, identifiers for different times considered (reporting time, response time), and important factors identified (classified as incident characteristics, traffic conditions, roadway, environmental characteristics and others).

Current Study in Context

It is evident that previous research has provided remarkable findings on total incident duration, clearance time, and response time. However, limitations still exist in earlier work. The examination of all components of incident duration as dependent variables is critical to understanding the overall incident clearance process. The consideration of components such as reporting time as explanatory variable in modeling other durations could potentially lead to endogeneity bias. The factors that increase reporting time (such as an accident in a remote location) could potentially lead to increased response time and clearance time. Thus, the model development process for incident duration should examine the impact of various components of incident duration within a multivariate framework. The current study develops a tri-variate framework that accommodates for inherent dependencies across the various components of incident duration. Second, a unique ordered response model structure (sometimes referred to as grouped ordered response model) is introduced for modeling the three durations in our study. In the presence of large groupings of dependent variables around a particular value a linear regression model (or a log-linear model) would perform poorly. The same drawback will apply to hazard duration models. On the other hand, our approach by allowing the grouped alternatives reduces the sensitivity of the model to large groupings around a single value. The proposed grouped ordered response approach is the first application of this model for incident duration analysis.

Finally, as opposed to employing a simulation oriented multivariate model approach, we propose and estimate a copula based methodology that allows for a closed-form probability computation. In recent years, several studies have highlighted the values of copula models in transportation ((*9*), (*10*), (*11*), (*12*)). The proposed copula framework is estimated to identify factors affecting incident duration components from a host of characteristics including incident characteristics, traffic conditions, roadway, and environmental characteristics. The data for the analysis is obtained from the 2015 FDOT incident database for events involving road ranger professionals. A model illustration exercise is conducted to highlight potential applications of the proposed model using 3D surface plots. A validation exercise is also conducted to test the predictive performance of the model.

TABLE 1 Summary of Literature Review

Study	Study Region and Data Source	Outcome Variable	Type of Model	Reporting Time Considered?	Response Time Considered?	Important Factors Identified			
						Incident	Traffic	Roadway	Environmental
Golob et al., (*13*)	Los Angeles / 1983-1985	Total incident duration	Log-normal Distribution	No	No	Yes	No	No	No
Giuliano, (*14*)	Los Angeles / 1987	Freq. and Total incident duration	ANOVA	No	No	Yes	Yes	No	No
Nam and Mannering, (*15*)	Washington / 1994 and 1995	Reporting, Response, and Clearance Time	HBM: Weibull and Log-Logistic Survival	Yes	Yes	Yes	Yes	Yes	Yes
Chimba et al., (*16*)	Tennessee / 2004 - 2010	Total incident duration	HBM: Log-Logistic Survival Model	No	No	Yes	Yes	Yes	Yes
Hojati et al., (*17*)	Queensland, Australia / 2009	Log of Incident Duration in minutes	HBM: Log-Logistic, Lognormal, Weibull Survival	No	No	Yes	Yes	Yes	Yes
Garib et al. Radwan, Al-Deek, (*18*)	Oakland, California / 1993	Delay in vehicle hours and Log of Incident Duration	Multiple Regression Models	No	No	Yes	Yes	Yes	Yes
Ding et al., (*19*)	Washington / 2009	Response Time and Clearance Time	Binary probit and Switching regression models	No	Yes	Yes	Yes	No	Yes
Weng et al., (*20*)	Maryland I-95 / 2010 and 2011	Response Time and Clearance time	Cluster Based Lognormal Distribution Model	No	No	Yes	Yes	Yes	Yes
Khattak et al., (*21*)	Chicago / 1989-1990	Total incident duration	Truncated regression, Time sequential models	No	Yes	Yes	Yes	No	Yes
Valenti et al., (*22*)	Italy / 2005 (3 Months)	Total incident duration	L. Regression, Decision Tree, ANN, Support Vector Machine (SVM), kNN	No	No	Yes	Yes	Yes	No

Smith and Smith, (*23*)	Virginia, I-64, I-264 / 1997-2000	Clearance Time	Non-parametric Regression and CART,	Yes	Yes	Yes	No	No	Yes
Demiroluk and Ozbay, (*24*)	New Jersey / 2005	Total incident duration	Three types of Bayesian Networks: Naïve Bayes, TAN, and K2	No	No	Yes	Yes	Yes	Yes
Chung, (*25*)	S. Korea / 2006 and 2007	Total incident duration	Hazard Based Duration Models (HBM): Log-Logistic AFT	No	No	Yes	Yes	Yes	No
Sullivan, (*26*)	Chicago, Charlotte, Houston, Orlando, SF, LA/ 1994	Total incident duration	Log-normal Distribution and IMPACT model	Yes	Yes	Yes	Yes	Yes	No
Hu et al., (*27*)	London / 2007-2008	Total incident duration	HBM: Log-logistic AFT Survival Model	No	No	Yes	Yes	No	Yes
Wang et al., (*28*)	United Kingdom / 2000-2001	Total incident duration	Fuzzy Logic and ANN	No	No	Yes	Yes	Yes	No
Ozbay and Noyan, (*29*)	N. Virginia / 1997	Clearance Time	Bayesian Networks	No	No	Yes	No	Yes	No
Lee and Wei, (*30*)	Taiwan / 2004-2005	Response time and Clearance time	Genetic Algorithm and ANN models	No	No	Yes	Yes	Yes	Yes
Wu et al., (*31*)	Netherlands, 2005	Total incident duration	Support Vector Regression (SVR)	No	No	Yes	Yes	No	Yes
Zhan et al., (*32*)	F. Lauderdale / 2006-2007	Clearance Time	M5P Tree Algorithm	No	Yes	Yes	Yes	Yes	Yes
Khattak and Wang, (*33*)	Hampton / 2006	Total incident duration	OLS and truncated regression	No	No	Yes	Yes	Yes	Yes
Zhang and Khattak, (*34*)	Hampton Roads / 2005	Contained and Extended event durations	OLS Regression	No	No	Yes	Yes	Yes	Yes

Ghosh et al., (*4*)	Michigan / 2009	Clearance Time	HBM: Exp. Weibull, Lognormal, Log-logistic, Gamma	No	No	Yes	Yes	Yes	Yes
Chung and Yoon, (*35*)	California / 2001	Total incident duration	HBM: Lognormal AFT	No	No	Yes	Yes	Yes	Yes
Kaabi et al., (*36*)	Abu Dhabi, UAE / 2009	Response Time	HBM: Weibull, Lognormal, Log-Logistic AFT	No	Yes	Yes	Yes	Yes	Yes
Vlahogianni, (*37*)	Athens, Greece / 2012	Total incident duration	Partial Logistic Regression ANN	No	No	Yes	Yes	Yes	Yes
Pereira et al., (*8*)	Singapore / 2010-2011	Total incident duration	LDA Topic Modelling / LR, SVM, ANN, DT	No	No	Yes	Yes	Yes	Yes
Hojati et al., (*38*)	Queensland 2011	Incident duration, Recovery Time	HBM: Weibull, Log-logistic AFT	No	No	Yes	Yes	Yes	Yes
Li, (*39*)	Beijing, China / 2008	Preparation, Travel, Clearance Time	HBM: Gamma, Weibull AFT	No	Yes	Yes	Yes	No	Yes
Zou et al., (*40*)	Washington I-5 / 2009	Clearance Time	HBM: Weibull, Log-logistic, Lognormal AFT	No	Yes	Yes	Yes	Yes	Yes
Ma et al., (*6*)	Washington I-5 / 2012	Clearance Time	Gradient Boosting Decision Tree	No	Yes	Yes	Yes	Yes	Yes
Zhu et al., (*41*)	Washington / 2011-2013	Incident Duration	kNN / HBM: Log-logistic, lognormal, Weibull AFT	No	No	Yes	Yes	No	No

ECONOMETRIC METHODOLOGY

Copula Based Approach

A copula is a device or function that generates a stochastic dependence relationship (*i.e.*, a multivariate distribution) among random variables with pre-specified marginal distributions (*9*), (*42*). Let C be an I-dimensional copula of uniformly distributed random variables $U_1, U_2, U_3, \dots, U_I$ with support contained in $[0,1]^I$. Thus, the joint distribution of these random terms can be defined as:

$$C_\theta(u_1, u_2, u_3, \dots, u_\mathrm{I}) = Pr(U_1 < u_1, U_2 < u_2, U_3 < u_3, \dots, U_I < u_I) \quad (1)$$

where θ is a parameter vector of the copula commonly referred to as the dependence parameter vector. Let us consider I $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_\mathrm{I})$ random variables each with univariate continuous marginal distribution function $F(Z_i) = \Pr(\varepsilon_i < Z_i)$. Thus, a joint I-dimensional distribution function of the random variables with the continuous marginal distribution functions $F(Z_i)$ can be formed as follows (*43*):

$$\begin{aligned} F(Z_1, Z_2, Z_3, \dots, Z_I) &= \Pr(\varepsilon_1 < Z_1, \varepsilon_2 < Z_2, \varepsilon_3 < Z_3, \dots, \varepsilon_I < Z_I) \\ &= C_\theta[u_1 = F(Z_1), u_2 = F(Z_2), u_3 = F(Z_3), \dots, u_\mathrm{I} = F(Z_1)) \end{aligned} \quad (2)$$

The specification defined in equation 2 offers an approach to develop different dependency patterns for the random variables $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_\mathrm{I})$ based on the copula that is used as the underlying basis of construction.

Ordered Group Response Framework

Let q be an index for events (incidents in the current empirical context) $(q = 1, 2, \dots, Q)$, and let j be the index for duration components $(j = 1, 2, \dots, J)$. Also, let k_j be an index for the time intervals $(k_j = 1, 2, \dots, K_j)$ for the duration components. In our study, j takes the values of reporting time (j = 1), response time (j = 2) and clearance time (j = 3). In the usual ordered response framework notation, one can write the latent propensity (y^*_{qj}) of duration component j for incident q to take a time interval level as a function of relevant covariates, and then relate this latent propensity to the time interval (y_{qj}) representing the time interval elapsed for duration component j in the event of incident q through threshold bounds:

$$y^*_{qj} = \boldsymbol{\beta}_j \boldsymbol{x}_{qj} + \boldsymbol{\sigma}_{k_j} + \varepsilon_{qj}\,, \;\; y_{qj} = k_j, if\ \psi_{k_j} < \; y^*_{qj} < \psi_{k_j+1} \quad (3)$$

where $\boldsymbol{x}_{qj}$ is a vector of exogenous variables for duration component j in incident q, $\boldsymbol{\beta}_j$ is a corresponding vector of coefficients to be estimated, and ψ_{k_j} is the lower bound threshold for grouped time interval level k_j specific to duration component j. The ε_{qj} terms capture the idiosyncratic effect of all omitted variables for duration component j for the event q. Further, $\boldsymbol{\sigma}_{k_j}$ is a vector of time interval category-specific coefficients for time interval alternative k_j in duration component j. The ε_{qj} terms are assumed identical across duration components, each with a univariate continuous marginal distribution function $F(Z_{qj}) = \Pr(\varepsilon_{qj} < Z_{qj})$. The error terms are assumed to be independently logistic distributed with variance $\boldsymbol{\lambda}_{qj}^2$. The variance vector is parameterized as follows:

$$\boldsymbol{\lambda}_{qj} = exp(\delta + \boldsymbol{\varrho}\boldsymbol{w}_{qj}) \tag{4}$$

where, δ is a constant, $\boldsymbol{w}_{qj}$ is a set of exogenous variables associated with duration component j in incident q and $\boldsymbol{\varrho}$ is the corresponding vector of parameters to be estimated. The parameterization allows for the variance to be different across events accommodating for heteroscedasticity. Given these relationships across the different parameters, the probability for duration component j for time interval in category k_j is given by:

$$Pr(y_{qj} = k_j) = \phi\left(\frac{\psi_{k_j+1} - (\boldsymbol{\beta}_j\boldsymbol{x}_{qj} + \boldsymbol{\sigma}_{k_j})}{\boldsymbol{\lambda}_{qj}}\right) - \phi\left(\frac{\psi_{k_j} - (\boldsymbol{\beta}_j\boldsymbol{x}_{qj} + \boldsymbol{\sigma}_{k_j})}{\boldsymbol{\lambda}_{qj}}\right) \tag{5}$$

where, $\phi(\cdot)$ is the standard logistic distribution function.

Joint Model Formulation and Estimation

In examining the grouped time intervals across different duration components simultaneously, the levels of correlations between three dimensions of interests depend on the type and extent of dependency among the stochastic terms (ε_{qj}) of equations 3. Thus, dependence in the ε_{qj} terms across duration components j in the same event q is accommodated to allow for the unobserved cluster effects and the copula based joint probability function as presented in equation 2 can be expressed as:

$$\begin{aligned} F(Z_{q1}, Z_{q2}, Z_{q3}) &= \Pr(\varepsilon_{q1} < Z_{q1}, \varepsilon_{q2} < Z_{q2}, \varepsilon_{q3} < Z_{q3}) \\ &= C_{\theta_q}[u_{q1} = F(Z_{q1}), u_{q2} = F(Z_{q2}), u_{q3} = F(Z_{q3}))\end{aligned} \tag{6}$$

where, $C_{\theta_q}$is the copula density. It is important to note here that, the level of dependence between grouped time interval levels across different duration components can vary across clusters. Therefore, in the current study, the dependence parameter θ_q is parameterized as a function of observed attributes as follows:

$$\theta_q = fn(\boldsymbol{\gamma}\boldsymbol{s}_q) \tag{7}$$

where, $\boldsymbol{s}_q$ is a column vector of exogenous variables, $\boldsymbol{\gamma}$ is a vector of unknown parameters (including a constant) and fn represents the functional form of parameterization. Based on the dependency parameter permissible ranges, alternate parameterization forms for the four copulas are considered in our analysis. For the Clayton and Frank copulas we employ $\theta_q = exp(\boldsymbol{\gamma}\boldsymbol{s}_q)$, and for Joe and Gumbel copulas we employ $\theta_q = 1 + exp(\boldsymbol{\gamma}\boldsymbol{s}_q)$ (see (*44*); (*45*); (*46*) for a similar approach).

Thus, the probability of the observed vector of grouped time interval levels across different duration components in incident q can be written as:

$$Pr\left(y_{q1}=k_{q1},y_{q2}=k_{q2},y_{q3}=k_{q3}\right) = \int_{K_q} C_{\theta_q}\left(F(y_{q1}^*),F(y_{q2}^*),F(y_{q3}^*)\right) dy_{q1}^* dy_{q2}^* dy_{q3}^* \quad (8)$$

where, the integration domain K_q is simply the multivariate region of the y_{qj}^* variables determined by the observed vector of grouped time interval variables (k_{q1},k_{q2},k_{q3}). The dimensionality of the integration, in general, is equal to the number of duration components in the incident. In our current study context, we consider event-level cluster with identical dependencies between pairs of duration components in the incident. The cluster size in our study is 3 and allows us to estimate the formulation presented in equation 6 in a closed form structure (see (*47*) for detailed discussion of computation issue with clusters greater than 5). The probability in equation 6 can be written in terms of 2^3 closed-form multivariate cumulative distribution functions as follows:

$$\begin{aligned}
&Pr\left(y_{q1}=k_{q1},y_{q2}=k_{q2},y_{q3}=k_{q3}\right) \\
&= Pr(\psi_{k_{q1}} < y_{q1}^* < \psi_{k_{q1}+1}, \psi_{k_{q2}} < y_{q2}^* < \psi_{k_{q2}+1}, \psi_{k_{q1}} < y_{q3}^* < \psi_{k_{q3}+1}) \\
&= \sum_{a_1=1}^{2}\sum_{a_2=1}^{2}\sum_{a_3=1}^{2}(-1)^{a_1+a_2+a_3}\left[Pr(y_{q1}^* < \psi_{k_{q1}+a_1-1}, y_{q2}^* < \psi_{k_{q2}+a_2-1}, y_{q3}^* < \psi_{k_{q3}+a_3-1})\right] \quad (9) \\
&= \sum_{a_1=1}^{2}\sum_{a_2=1}^{2}\sum_{a_3=1}^{2}(-1)^{a_1+a_2+a_3}\left[C_{\theta_q}(u_{k_{q1}+a_1-1}, u_{k_{q2}+a_2-1}, u_{k_{q3}+a_3-1})\right]
\end{aligned}$$

The parameters to be estimated in the model may be gathered in a vector $\Omega = (\boldsymbol{\beta}_j, \boldsymbol{\sigma}_{k_j}, \psi_{k_j}, \delta, \boldsymbol{\varrho}, \boldsymbol{\gamma})$. The likelihood function for incident q may be constructed based on the probability expression in equation 9 as:

$$L_q(\Omega) = Pr\left(y_{q1}=k_{q1},y_{q2}=k_{q2},y_{q3}=k_{q3}\right) \quad (10)$$

The likelihood function to be maximized is then given by:

$$L(\Omega) = \prod_q L_q(\Omega) \quad (11)$$

In our analysis we employ four Archimedean copulas Frank, Clayton, Joe and Gumbel copulas (a detailed discussion of these copulas is available in (*9*)). The estimation routines for the models in current study are coded in GAUSS Matrix Programming software (see (*48*)).

DATA DESCRIPTION

In this paper, we focus on Central Florida traffic incidents provided by FDOT that occurred in year 2015 with participation from at least one Road Ranger patrol. In 2015; Road Rangers responded to a total of 57,238 incidents. Through an extensive data preparation process, only those events in which complete information was available for reporting time, response time, and clearance time were selected. Subsequently, the final compiled dataset consisted of 50,319 events. From the final dataset, 14,870 records are randomly sampled for the purpose of model estimation.

First row panel of Table 2 presents the dependent variables statistics considered in current study context. We consider five, eight and eleven categories of reporting, response and clearance time, respectively. With respect to independent variables, the attributes considered can be grouped into four broad categories: Incident characteristics, Traffic characteristics, Roadway characteristics and Environmental characteristics. Descriptive statistics of variables considered in our modeling exercise are provided in second row panel of Table 2.

TABLE 2 Sample Statistics

Variables Name	**Definition**	**Statistics**		
DEPENDENT VARIABLES				
Attributes		**Min.**	**Max.**	**Average**
System Level Characteristics				
Reporting Time	*Notified Time – Detected Time (minutes)*	0.00	244.21	0.13
Response Time	*Arrival Time - Notified Time (minutes)*	0.00	392.93	1.79
Clearance Time	*Closed Time - Arrival Time (minutes)*	0.00	8924.00	81.81
Group-specific Characteristics				
Attributes		**Frequency**		
Reporting time				
T1-Bin 1	>0~0.5 *minutes*	14551		
T1-Bin 2	>0.5~1 *minutes*	134		
T1-Bin 3	>1~1.5 *minutes*	52		
T1-Bin 4	>1.5~2 *minutes*	22		
T1-Bin 5	>2 *minutes*	111		
Response time				
T2-Bin1	>0~5 *minutes*	13828		
T2-Bin2	>5~10 *minutes*	201		
T2-Bin3	>10~15 *minutes*	206		
T2-Bin4	>15~20 *minutes*	153		
T2-Bin5	>20~30 *minutes*	200		
T2-Bin6	>30~40 *minutes*	127		
T2-Bin7	>40~50 *minutes*	57		
T2-Bin8	>50 *minutes*	98		
Clearance time				
T3-Bin1	0~5 *minutes*	6469		
T3-Bin2	>5~10 *minutes*	2096		
T3-Bin3	>10~15 *minutes*	1334		
T3-Bin4	>15~20 *minutes*	961		
T3-Bin5	>20~40 *minutes*	1526		
T3-Bin6	>40~60 *minutes*	592		
T3-Bin7	>60~80 *minutes*	291		
T3-Bin8	>80~100 *minutes*	226		
T3-Bin9	>80~120 *minutes*	140		

T3-Bin10	>120~140 *minutes*	98
T3-Bin11	>140 *minutes*	1137
INDEPENDENT VARIABLES		
Continuous Predictors	**Min./Max.**	**Average**
Distance from CBD	0.003/0.278	0.078
Categorical Predictors	**Frequency**	**Percentage (%)**
Incident Characteristics / Activity Type		
Number of vehicles involved		
More than one vehicles	487	3.3
One vehicle	14383	96.7
Event type		
Crash	1102	7.4
Disabled vehicle	8683	58.4
Other event	5,085	34.2
Activity type		
Tire Service	2304	15.5
Abandoned Activity	1348	9.1
Debris Activity	3498	23.5
Mechanical Activity	1997	13.4
Other activity	5723	38.5
Notifier agency		
Notified by Police	828	5.6
Notified by Road Ranger	1254	8.4
Other agency	12788	86
RR agency responded		
I-4 Road Ranger responded	6315	42.5
CFX Road Ranger responded	7908	53.2
Other RR agency	647	4.3
Traffic Characteristics		
Time of the day		
Morning peak	3782	25.4
Evening peak	4432	29.8
Nighttime	1720	11.6
Day of the week		
Weekday	10,725	72.1
Weekend	4145	27.9
Roadway Characteristics		
Event location		
Intersection	4922	33.1
Non-intersection	9948	66.9
Built environment		
Rural	1511	10.2
Urban	13359	89.8
County		
Seminole	1269	8.5
Other counties	13,601	91.5
Roadways		
At Interstate-4	6691	45

At State Road 417	1682	11.3
At State Road 429	1210	8.1
At State Road 408	3338	22.4
Other roadways	1,949	13.2
Environmental characteristics		
Months		
July	1379	9.3
August/September	1269	8.5
September	1197	8
Other months	12,222	82.2

EMPIRICAL ANALYSIS

Optimal Copula Model Selection

The empirical analysis involves four different families of copula models estimation to explain the dependence between reporting, response, and clearance times of traffic incidents. An independent copula model (separate ordered group response models for incident durations) is estimated to establish a benchmark for copula structure selection. The copula models estimated in our analysis include: 1) Clayton, 2) Gumbel, 3) Frank and 4) Joe. For each of the selected copula structures, models with and without parameterization in the dependence effects were also estimated (see Equation 7). Thus, a comparison of 9 model estimations was undertaken. In order to determine the optimal copula model, performances of the alternative copula models were tested by log-likelihood values as well as employing the Bayesian Information Criteria (BIC). The copula with the lowest BIC is the preferred model. The BIC values for the various estimated models are presented in Table 3. Please note that the Clayton copula models collapsed to the independent model (thus instead of 9 rows, we have only 7 rows). Based on the model fit comparison, several copula models offer improved fit relative to the independent model supporting our hypothesis that those three components of incident duration exhibit strong dependency. Among the copula models, the Gumbel parameterized model offers the most significant improvement in log-likelihood and lower BIC. Henceforth, the Gumbel parameterized model structure is described in detail.

TABLE 3 Log-Likelihood and BIC of Copula Models

Models	Number of Parameters	Log-likelihood	BIC
Gumbel-Parameterized	55	-33045.90	*66620.19*
Joe-Parameterized	55	-33060.92	66650.23
Frank-Parameterized	55	-33081.59	66691.56
Gumbel	53	-33108.20	66725.58
Frank	53	-33111.92	66733.02
Joe	53	-33129.62	66768.41
Independent	56	-33152.22	66842.44

Estimation Results

In this section, the results for the Gumbel-Parameterized copula based model are presented. In Table 4, the first, second and third column panels of results correspond to the reporting, response and clearance time. For the ease of model discussion, the impact of an explanatory variable is presented for all three components of the joint model at the same time. A positive (negative) coefficient associated with a variable indicates propensity for longer (shorter) duration.

Incident Characteristics

The results highlight a variation in duration based on event type. Specifically, major incidents such as incidents involving multiple vehicle or crash events tend to have longer response times (*15*). On the other hand, minor incidents such as disabled vehicles have lower response and clearance times. Abandoned vehicle and debris events are found to be associated with shorter response delays compared to other activities. Tire services such as a tire change leads to longer response time. Mechanical activity associated events (such as engine, gas, overheating activities) are found to be significantly associated with shorter clearance time.

In terms of notification personnel, if the incident is notified by a police officer, reporting time reduces significantly. According to the scale component results; when the notifier is a Road Ranger, incident clearance time increases. Road Ranger patrols are categorized as I-4, Broward, Palm Beach and Central Florida Expressway Authority (CFX) Road Rangers. Among these categories, I-4 Road Rangers as well as CFX Road Rangers are associated with longer response delays compared to other Road Rangers. It is possible that the presence of heavy traffic on these facilities increases response times.

Traffic Characteristics

With respect to traffic characteristics, model results show a significant reduction in reporting time of incidents occurring during morning peak hours (see (*15*) for a similar result). On the other hand, response time tends to increase in evening rush hours. The results in the literature are ambiguous in this regard with supporting as well as contradictory evidence ((*15*) and (*49*)). The increase in response time can be attributed to increased travel delay resulting in longer travel times for responders. Clearance time is likely to increase in both morning and evening peak hours. Previous research is consistent with this finding (*15*). Response and clearance times of nighttime (10:00pm to 6:00am) incidents tend to take longer compared to response and clearance time of daytime (6:00am to 10:00pm) incidents. Traffic incidents occurring on weekends have a tendency to reduce incident clearance time as compared to weekday incidents which is consistent with the findings from earlier literature ((*50*), (*51*)).

Roadway Characteristics

It can be observed from the results that location of crashes has significant effects on the incident duration phases. Reporting and response time for events that occur within Seminole County was found to take longer than other counties. Based on data from FDOT crash statistics Seminole county is one of the safest counties in the state (*52*). Thus, the resource allocation of the county towards incident management might be lower than other counties. Furthermore, the responder agencies in the vicinity of Seminole county are I-4 road rangers and CFX road rangers. Given that these major roadways are further from Seminole county relative to other counties could potentially increase reporting and response times. Moreover, among the highways analyzed in our model, incidents that occurred on Interstate highway-4 (I-4) tend to have longer response and clearance times. The significant traffic volume on the roadway is a plausible reason for these findings.

Central Florida region is home to several tollways. Among these, tollway state road (SR) 417 incidents are found to be associated with longer clearance time, whereas toll road SR 429 incidents show significant association with shorter clearance time. The results are intuitive given the traffic volumes observed on these facilities. SR-417 is a 55 mile stretch of tollway with about 375 thousand vehicles served per day. SR-429 is a 23mile stretch of tollway with about 123 thousand vehicles served per day. Thus responding personnel have easier access for SR 429 reducing clearance time. Traffic events at intersections take longer reporting, response, and clearance times. The distance from incident location to the Central Business District (CBD) area was considered in the model. According to the results, longer distances for CBD tended to have longer response delays and clearance durations. This finding is consistent with a study effort which indicates that longer distances from CBD tend to increase the total incident duration (*17*).

Environmental Characteristics

Among the months considered in the models, summer months were found to significantly influence the reporting and response times. For example, when incidents occur in July, August or September; reporting and response times reduce relative to reporting and response times for all other months. The reasons for the differential impacts in these months is not immediately apparent. It is possible that during these months, traffic volumes in Central Florida region are lower as the universities and schools are closed. The result warrants future investigation.

Alternative Specific Effects

In the grouped ordered specification of the joint model, we also estimate alternative specific constants for categories considered across different duration components. It is worthwhile to mention here that it is possible to estimate group-specific effects for each group considered across different duration components. However, in our joint model specifications, we estimate group-specific effects if it improves data fit. The results of these group specific effects are presented in second row panel of Table 4. For reporting time component, none of the group-specific effects improved data fit further and hence are not included in our final model specifications. With respect to response and clearance time, group-specific components are estimated for one (T2-Bin2) and four (T3-Bin1, T3-Bin2, T3-Bin3 and T3-Bin4) categories, respectively. Adding more group-specific components did not improve the data fit further in the current study context and hence are not included in our final joint model specifications. These parameters are similar to constants in discrete choice models and do not really have a substantive interpretation.

Variance Component

As indicated earlier, the variance of the grouped ordered components are estimated as a function of observed exogenous variables in current study context. The parameter estimates of these components are presented in the third-row panel of Table 4. From Table 4, we can see that the variance component of the grouped ordered models are characterized by several exogenous variables. The exogenous variables that contribute to the variance profile of reporting time model include notified by police, I-4 road ranger responded and CFX road ranger responded. The exogenous variables that contribute to the variance profile of response time component of the joint model include at intersection and disabled vehicle. Finally, at rural area, at SR 408 and notified by road ranger indicator variables are found to have significant effect on the variance profile of clearance time model. Overall, these models results illustrate the presence and the magnitude of heteroscedasticty in our data.

Dependence Effects

The estimation results of the dependence effects are presented in last row panel of Table 4. Gumbel copula offers an asymmetric dependency structure and the dependency is entirely positive. The coefficient sign and magnitude reflects whether a variable increase or reduces the dependency and by how much. The dependence results highlight the presence of common unobserved factors affecting different duration components. Further, from the estimated results we can see that the copula dependencies are characterized by additional exogenous variables. This provides support to our hypothesis that the dependency structure is not the same across the sample population. The various exogenous variables that contribute to the dependency include crash, abandoned activity, debris activity, tire service and disabled vehicle.

TABLE 4 Gumbel Parameterized Copula Model Results

Gumbel-Parameterized Copula Ordered Logit - Generalized Ordered Logit Model						
Variables	**Reporting Time**		**Response Time**		**Clearance Time**	
	Estimate	**t-stat.**	**Estimate**	**t-stat.**	**Estimate**	**t-stat.**
Constant	-3.975	-13.28	-37.502	-15.18	-141.212	-16.31
Incident Characteristics / Activity Type						
Number of vehicles involved (Base: One vehicle)						
More than one vehicles	0.828	4.56	-	-	-	-
Event type (Base: Other event)						
Crash	2.146	11.13	-	-	-	-
Disabled vehicle	-	-	-46.768	-14.32	-36.978	-10.55
Activity type (Base: Other activity)						
Tire Service	-	-	5.96	2.62	-	-
Abandoned Activity	-	-	-45.542	-9.34	-	-
Debris Activity	-	-	-13.885	-7.78	-	-
Mechanical Activity	-	-	-	-	-34.652	-6.73
Notifier agency (Base: Other agency)						
Notified by Police	-13.671	-2.1	-	-	-	-
Notified by Road Ranger	-	-	-	-	-17.161	-3.09
RR agency responded (Base: Other RR agency)						
I-4 Road Ranger responded						
CFX Road Ranger responded						
Traffic Characteristics						
Time of the day (Base: Other time)						
Morning peak	-0.477	-2.37	-	-	21.035	5.16
Evening peak	-	-	9.134	6.57	19.787	4.11
Nighttime	-	-	7.397	4.43	48.511	9.3
Day of the week (Base: Weekdays)						
Weekend	-	-	-	-	-8.677	-2.7
Roadway Characteristics						
Distance from CBD	-	-	0.005	3.61	0.015	5.17
Event location (Base: Non-intersection)						

Intersection	0.509	3.2	6.549	3.24	12.135	3.3
County (Base: Other counties)						
Seminole	0.67	3.28	5.397	2.97	-	-
Roadways (Base: Other roadways and State 408)						
At Interstate-4	-	-	22.694	13.37	52.956	12.24
At State Road 417	-	-	-	-	32.706	6.31
At State Road 429	-	-	-	-	-39.833	-6.42
Environmental characteristics						
Months (Base: Other months)						
July	-0.485	-1.65	-6.366	-2.86	-	-
August/September	-0.473	-2.28	-	-	-	-
September	-	-	-6.898	-2.91	-	-
Variance Components						
Constant	-0.273	-2.74	2.771	69.24	4.248	111.11
At Intersection	-	-	-0.092	-2.12	-	-
At Rural Area (Base: Urban)	-	-	-	-	-0.106	-3.51
At State Road 408	-	-	-	-	0.182	6.88
Disabled Vehicle	-	-	0.414	7.64	-	-
Notified by Police	1.168	2.93	-	-	-	-
Notified by Road Ranger	-	-	-	-	0.405	12.23
I-4 Road Ranger responded	0.584	6.69	-	-	-	-
CFX Road Ranger responded	0.229	2.62	-	-	-	-
Dependence Effects						
Variables	**Estimate**			**t-stat.**		
Constant	-1.6			-15.971		
Abandoned Activity	-3.309			-3.65		
Debris Activity	0.688			5.58		

Model Illustration and Validation Analyses

To demonstrate the implications of the estimated model, we apply the developed model to generate response surface with respect to reporting time, response time, clearance time and incident frequencies. In generating the values for plotting the response surface, we identify the incident duration categories based on probabilistic assignment by using predicted probabilities computed from the best specified copula model (Gumbel parameterized). The probabilities are appropriately aggregated across categories to identify the corresponding frequencies. For illustration purposes, we plot the response surfaces for events on I-4 and other roadways across different time periods (morning peak and evening peak). In Figure 1, predicted incident frequencies (Z-axis) identified based on probabilistic assignments are depicted by 3-dimensional charts as a function of the time ranges of reporting/response time and the clearance time categories. Specifically, the X-axis include 4 outcomes – combination of 2 levels of reporting time (≤ 0.5 minutes and > 0.5 minutes) and 2 levels of response time (≤ 5 minutes and > 5 minutes). The Y-axis represents the clearance time. The reader would note that the plots provided are only a sample of the various illustrations that can be generated based on the independent variables in the models. Overall, from the figure we can observe that incident duration is higher for any time period on I-4 relative to incidents on other roadway locations. Incidents during morning peak result in longer incident durations than incidents in evening peak period as illustrated in Figure 1. The development of such response surface could be helpful for the incident management agencies to allocate their resources based on the reported incident scenarios.

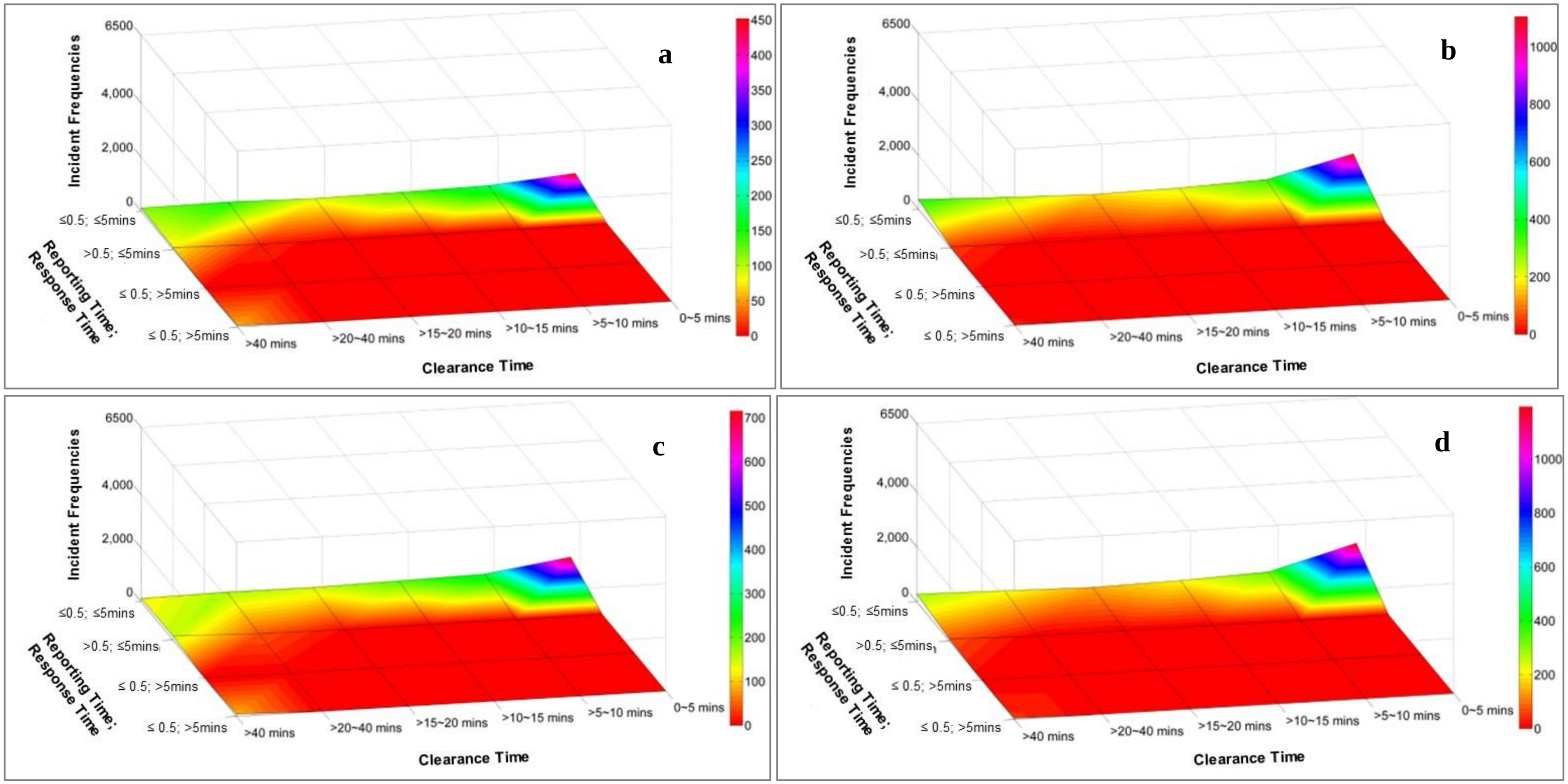

FIGURE 1 Response Category Surfaces of Predicted Events
(a) I-4 and Morning Peak Period, (b) I-4 and Evening Peak Period, (c) Roadways other than I-4 and Morning Peak Period, (d) Roadways other than I-4 and Evening Peak Period

CONCLUSION

To understand the overall incident clearance process, this paper formulated and estimated a copula based tri-variate framework accommodating for inherent dependencies across the three components of incident duration - reporting, response, and clearance time. To the best of the authors' knowledge, this is the first attempt to employ a tri-variate copula based methodology in incident duration analysis. Moreover, the study contributes to the incident management literature by examining factors affecting the clearance process including incident duration components such as incident characteristics, traffic conditions, roadway, and environmental characteristics. The empirical analysis involves estimation of models using four different copula structures: 1) Clayton, 2) Gumbel, 3) Frank and 4) Joe. The comparison between copula and the independent models, based on information criterion metrics, confirmed the importance of accommodating dependence across reporting time, response time, and clearance time in incident duration analysis. The most suitable copula model is obtained for Gumbel copula with parameterization for dependence profile. The model estimates were also augmented by conducting policy analysis and 3-dimensional representation of incident frequencies as a function of reporting, response, and clearance time.

The reader would note that the proposed copula approach can be employed to consider hazard duration and/or linear regression based model structures. However, if the data has a large share of observations concentrated around a single value (nearer to 0 in our study) hazard duration and linear regression models are likely to perform poorly. The proposed approach is not as affected by the large concentrations around a single value. Finally, the proposed model structure does not accommodate for potential endogeneity between the three dependent variables. The consideration for endogeneity is an avenue for future research.

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