# A Copula Based Joint Model of Injury Severity and Vehicle Damage in Two-Vehicle Crashes

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**ABSTRACT**

In the transportation safety arena, in an effort to improve safety, statistical models are developed to identify different factors that contribute to crashes, as well as various factors that affect injury severity in the unfortunate event of a crash. Our study contributes to the literature on severity analysis. Injury severity and vehicle damage are two important indicators of assessing severity in crashes. Typically injury severity and vehicle damage indicators are modeled independently. However, there are common observed and unobserved factors affecting the two crash indicators leading to potential interrelationships between them. Failing to account for the interrelationships of the indicators may lead to biased coefficient estimates in crash severity prediction models.

The focus of this study is to explore the interrelationships between the crash severity indicators: injury severity and vehicle damage, and also identify the nature of these correlations across different types of crashes. A copula based methodology that can simultaneously model injury severity and vehicle damage while also accounting for the interrelationships between the two indicators was employed in this study. Furthermore, parameterization of the copula structure was used to represent the interrelationships between the crash indicators as a function of the crash characteristics. In this study, six different specifications of the copula model including Gaussian, Farlie-Gumbel-Morgenstern (FGM), Frank, Clayton, Joe and Gumbel were developed. Based on goodness-of-fit statistics, the Gaussian copula model was found to outperform the other copula based model specifications. The results indicate that the interrelationships between injury severity and vehicle damage varied with different crash characteristics including manners of collision and collision types.

## BACKGROUND

Improving traffic safety was, is and will continue to be a high priority on the national transportation agenda due to the significant social and financial implications of motor vehicle crashes including injuries, deaths and economic losses among others. In the past few decades, organizations such as Federal Highway Administration (FHWA) *(1)*, American Association of State Highway and Transportation Official (AASHTO) *(2)* have launched numerous roadway safety campaigns and implemented various strategies for reducing the number of crashes with a particular emphasis on reducing the severe ones *(3)*. These efforts have been targeted at different aspects of roadway safety from improvements in highway engineering, to driver education, to driver assistance technologies and traffic enforcements. All of these efforts have led to a significant reduction in traffic fatalities from 43,510 in 2005 to about 32,367 in 2011(a 26 percent reduction in 7 year span) *(4)*. However, traffic safety still remains a significant externality and more needs to be done to alleviate the negative implications of crashes. In order to implement effective safety strategies and countermeasures, it is necessary to identify the different factors contributing to crashes and factors affecting crash severity in the event of a crash.

Injury severity is an important indicator that is usually modeled to identify the different factors contributing to driver injuries. Discrete choice methodologies have commonly been used to model the effects of driver, environmental, highway, traffic, and vehicle factors on injury severity *(5, 6)*. Among the different discrete choice methodologies, logistic and probit model formulations have been extensively used to examine the relationship between the contributing factors and injury severity. In studies where injury severity is treated as a non-ordinal indicator, the multinomial logistic or probit model formulations have been used to investigate the relationship between contributing factors and injury severity *(7-10)*. In studies where injury severity is treated as an ordinal variable, traditional ordered logistic or probit model formulations or generalized ordered logit formulations have been used *(11-21)*.

Both ordered and unordered logistic or probit models are fixed parameter models in which all parameters are assumed to be constant across observations. However, it is argued that model coefficients may not remain constant but vary across individuals when the data are heterogeneous. To this end, other model formulations were proposed to capture the heterogeneity across crashes. The Markov switching multinomial logistic model was used to account for unobserved factors that influence injury severity *(22)*. The random parameter (mixed) model is an alternative formulation which can treat the parameters as either fixed or random variables *(7, 9, 20, 23-27)*. More recently latent segmentation models that account for heterogeneity in a closed form structure in severity models have also been employed (*18*). Savolainen *et al. (28)* reviewed and summarized numerous discrete choice models that are currently being used in modeling injury severity and offered additional insights about model evaluation and selection.

Recently, in order to capture the interrelationships among variables when the factors interact in indirect and complicated ways in injury severity models, researchers have also extensively applied the structural equation modeling (SEM) in exploring the contribution of different explanatory variables on injury severity. SEM can effectively account for complex relationships between multiple dependent and independent variables simultaneously. Further, SEM can also incorporate the influence of latent variables on dependent variables of interest *(5, 6, 29-34)*.

Although injury severity has been used extensively in modeling the severity of a crash, it may not be the most representative indicator. Injury severity is a subjective indicator based on victim's responses, descriptions, and complaints after the crash; owing to the self-reported nature of the measure, it may be prone to bias *(6)*. On the other hand, the extent of vehicle damage is a more objective indicator based on the destruction/deformation of the vehicle involved in the crash; as it can be readily seen and measured. Due to its objective nature, vehicle damage has been used as an additional indicator to characterize crash severity *(6, 35-37)*.

Although vehicle damage has been introduced as an additional indicator in crash severity analysis, the treatment and modeling of the different indicators is up for debate. Injury severity and vehicle damage are typically modeled independently which may lead to possible estimation and inference issues because the two indicators are likely to be correlated (*35*). The levels of the indicators for any given crashes are correlated due to both observed and unobserved factors. Although the correlations due to the observed factors can be accounted for by specifying them as explanatory variables, same cannot be said about the unobserved factors because they are not observable. Ignoring the correlations due to unobserved factors may result in incorrect and biased coefficient estimates *(38)*. Therefore, there is a need for model formulations that can simultaneously model the injury severity and vehicle damage indicators of crash severity while also accounting for potential interrelationships between the indicators.

In this study, the copula based approach is used to model the injury severity and vehicle damage dimensions simultaneously while also accounting for the error correlations that may exist across the two dimensions. Further, in the copula approach, parameterization of the copula structure is allowed to help explain the heterogeneity in correlations between the dependent variables *(39)*. In recent years, the copula based model has been increasingly used in transportation research.

Pourabdollahi *et al. (40)* used a copula based model to estimate the choice of freight mode and shipment size simultaneously. The study confirms that the copula based model can effectively capture the effects of common unobserved factors affecting both variables, and consequently it can appropriately account for the correlations between the selection of freight mode and shipment size. Sener *et al. (41)* applied a copula based model to examine the physical activity participation for all individuals within the same family unit, by accounting for the dependencies among individuals' activity participation due to the common observed and unobserved factors. The model results show that individuals in the same family unit tend to have simultaneously low physical activity propensities, while the trend for high propensities is not significant.

The copula based model has also been used in modeling crash severity. Eluru *et al. (42)* examined the injury severities for all occupants involved in a crash using a copula based model. The effects of common unobserved factors on all occupants in the same vehicle were accommodated in the model. The results illustrate that the copula based model is better than the independent ordered probit model (in which the injury severity for each occupant was independently and separately modeled) with regard to the model goodness-of-fit. The study conducted by Rana *et al. (43)* employed a copula based model to consider the crash type and injury severity as dependent variables simultaneously. The model estimation results show that the copula based model outperforms the independent models in which the collision type and injury severity were independently modeled. Yasmin *et al. (39)* improved the model developed by Rana *et al.* by allowing the dependencies between injury severity and collision type to vary across different categories of collision type. The results suggest that injury severity and collision type are correlated, and the correlation between injury severity and collision type varies with the type of collision.

The research presented here is an attempt to model the injury severity and vehicle damage and to identify contributing factors, while also accounting for the potential correlations between the two indicators due to unobserved attributes. To this end, the copula based approach is applied to simultaneously model injury severity and vehicle damage. Given the ordered nature of the injury severity and vehicle damage indicators, ordered probit formulation was used to model both of the two indicators. The error correlations between the injury severity and vehicle were tied together using different copula formulations and parameterization strategies. The proposed model is estimated using the five-year (2005-2009) crash data for two-vehicle crashes collected from the Madison, Wisconsin, including a detailed set of exogenous variables, *i.e.*, driver characteristics, highway and traffic factors, environmental factors and crash characteristics. The rest of the paper is organized as follows. The next section presents the copula based methodology used in this paper. The third section describes the data in detail and the fourth section presents the model specifications and assumptions. The model results are presented in the fifth section, and concluding thoughts are presented in the final section.

## COPULA BASED MODEL

The primary objective of this study is to simultaneously model the injury severity and vehicle damage levels of crashes using a copula based approach. The indicators are treated as ordinal variables and a probit formulation is used to model the indicators. The econometric formulation of the proposed copula methodology is presented below:

### *Injury Severity Model Component*

Let $q$ $(q = 1,2,\dots,Q)$ be the index for vehicle involved in the crash, $j$ $(j = 1,2,\dots,J)$ be the index representing the level of injury severity and $k$ $(k = 1,2,\dots,K)$ be the index representing the level of vehicle damage. In an ordered probit formulation, the discrete injury severity level $(y_q)$ is assumed to be associated with an underlying continuous latent propensity $(y_q^*)$. Further, the latent propensity is specified as follows:

$$y_q^* = \alpha' x_q + \varepsilon_q\ ,\ \ y_q = j, if\ \tau_{j-1} < y_{qj}^* < \tau_j \tag{1}$$

where, $y_q^*$ is the latent propensity of injury severity for vehicle $q$, $x_q$ is a vector of exogenous variables, $\alpha$ is the associated row vector of unknown parameters and $\varepsilon_q$ is a random disturbance term assumed to be standard normal. $\tau_j$ $(\tau_0 = -\infty\ , \tau_J = \infty)$ represents the threshold associated with severity level $j$, with the following ordering conditions: $\left(-\infty < \tau_1 < \tau_2 < \dots < \tau_{J-1} < +\infty\right)$. Given the above information regarding the different parameters, the resulting probability expression for the occupant of vehicle $q$ sustaining an injury severity level $j$ takes the following form:

$$Pr(y_q = j) = \phi(\tau_j - \alpha' x_q) - \phi(\tau_{j-1} - \alpha' x_q) \tag{2}$$

where, $\phi(\cdot)$ is the standard normal distribution function. The probability expression in Equation 2 represents the independent injury severity model for the occupant of vehicle $q$.

***Vehicle Damage Model Component***

On the other hand, vehicle damage component also takes the form of an ordered probit formulation. The expression for latent propensity $(u_q^*)$ of vehicle damage is shown below:

$$u_q^* = \beta' z_q + \xi_q\ ,\ \ u_q = k, if\ \psi_{k-1} < u_{qk}^* < \psi_k \tag{3}$$

where, $u_q^*$ is the latent propensity of vehicle damage for vehicle $q$, $u_q$ is the discrete level of vehicle damage, $z_q$ is a vector of exogenous variables, $\beta$ is the associated row vector of unknown parameters, $\xi_q$ is a random disturbance term assumed to be standard normal and $\psi_k$ represents the threshold associated with vehicular damage level $k$. Assuming similar information for the thresholds as in the injury severity model component, the probability expressions for vehicle $q$ with a damage level $k$ can be written as:

$$Pr(u_q = k) = \Lambda(\psi_k - \beta' z_q) - \Lambda(\psi_{k-1} - \beta' z_q) \tag{4}$$

where, $\Lambda(\cdot)$ is the standard normal distribution function.

***Joint Model: A Copula based Approach***

In examining the injury severity and vehicle damage simultaneously, the dependency between the two dimensions of interests is captured through the error terms ($\varepsilon_q$ and $\xi_q$) from equation 1 and 3. The joint probability of sustaining injury severity level $j$ and vehicle damage level $k$ for vehicle $q$ can be expressed as:

$$\begin{aligned} Pr(y_q = j, u_q = k) \\ &= Pr\Big[\Big(\big(\tau_{j-1} - \alpha' x_q\big) < \varepsilon_q < \big(\tau_j - \alpha' x_q\big)\Big), \big((\psi_{k-1} - \beta' z_q) < \\ &\quad \xi_q < (\psi_k - \beta' z_q)\big)\Big] \\ &= Pr\big[\varepsilon_q < \big(\tau_j - \alpha' x_q\big),\ \ \xi_q < (\psi_k - \beta' z_q)\big] \\ &\quad - Pr\big[\varepsilon_q < \big(\tau_j - \alpha' x_q\big),\ \ \xi_q < (\psi_{k-1} - \beta' z_q)\big] \\ &\quad - Pr\big[\varepsilon_q < \big(\tau_{j-1} - \alpha' x_q\big),\ \ \xi_q < (\psi_k - \beta' z_q)\big] \\ &\quad + Pr\big[\varepsilon_q < \big(\tau_{j-1} - \alpha' x_q\big),\ \ \xi_q < (\psi_{k-1} - \beta' z_q)\big] \end{aligned} \tag{5}$$

Given the above setup, the correlations between the injury severity and vehicle damage due to unobserved factors are accommodated using a copula based approach. A detailed description of the copula approach can be found in Bhat and Eluru *(44)*, Trivedi and Zimmer *(45)*. The joint probability of equation 5 can be expressed by using the copula function as:

$$Pr\left(y_{qj}=j,u_{qk}=k\right)$$
$$=C_{\theta q}\left(U_{qj},U_{qk}\right)-C_{\theta q}\left(U_{qj},U_{qk-1}\right)-C_{\theta q}\left(U_{qj-1},U_{qk}\right)+C_{\theta q}\left(U_{qj-1},U_{qk-1}\right) \tag{6}$$

It is important to note here that the level of dependence between injury severity level and vehicle damage can vary across crashes. Therefore, in the current study, the dependence parameter $\theta_q$ is parameterized as a function of observed crash attributes as follows:

$$\theta_q=f_n(\gamma' s_q) \tag{7}$$

where, $s_q$ is a column vector of exogenous variables, $\gamma'$ is the associated row vector of unknown parameters (including a constant) and $f_n$ represents the functional form of parameterization. In this study, six different copula structures are respectively explored: Gaussian, Farlie-Gumbel-Morgenstern (FGM), Frank, Clayton, Joe and Gumbel copulas. A detailed discussion of these copulas is available in Bhat and Eluru *(44)*. Based on the permissible ranges of the dependency parameter, different functional forms are assumed for the parameterization of the six copula structures in the analysis. For Gaussian and Farlie-Gumbel-Morgenstern (FGM) copulas, functional form $\theta_q=\gamma' s_q$ is used. For the Clayton and Frank copulas, $\theta_q=exp(\gamma' s_q)$ is applied. Finally for Joe and Gumbel copulas, $\theta_q=1+exp(\gamma' s_q)$ is assumed. Further, similar parameterizations can be found in Sener *et al. (41)*, Eluru *et al. (42)* and Yasmin *et al. (39)*.

Of the six copulas, Clayton, Joe and Gumbel allow for asymmetric copulas that consider dependency in one direction. To potentially account for the possibility of a reverse dependency, with asymmetric copulas, a reverse dependent variable was considered for vehicle damage (wherein a new dependent variable is created by sorting vehicle damage from highest level to lowest level). This reversing of the dependent variables does not affect the ordered probit model probabilities (except for changes to the threshold values).

With the above as preliminaries, the likelihood function can be expressed as:

$$L=\prod_{q=1}^{Q}\left[\prod_{j=1}^{J}\prod_{k=1}^{K}\left\{Pr(y_q=j,u_q=k)\right\}^{\omega_{qkj}}\right] \tag{8}$$

where, $\omega_{qkj}$ is a dummy indicator variable assuming a value of 1 if injury severity level is $j$ and vehicle damage level is $k$ for the vehicle $q$ and 0 otherwise. All the parameters in the model are consistently estimated by maximizing the logarithmic function of $L$. The parameters to be estimated in the model are: $\alpha'$ and $\tau_j$ in the injury severity component, $\beta'$ and $\psi_k$ in vehicle damage component, and finally $\gamma'$ in the dependency component.

## DATA COLLECTION AND ANALYSIS

In this study, crash data collected in Madison, Wisconsin between 2005 and 2009 was used and only two-vehicle crashes were considered. Between 2005 and 2009, there were 13,683 two-vehicle crashes in Madison, Wisconsin, accounting for 60 percent of all crashes. Among all two-vehicle crashes, according to the Model Minimum Uniform Crash Criteria (MMUCC) guideline or "KABCO" scale *(46)*, 9,488 or 69.3 percent crashes were type O (no apparent injury or property damage only); 4,062 or 29.7 percent crashes were either type B (suspected minor injury) or C (possible injury); and 133 or 1 percent crashes were either type A (suspected serious injury) or K (fatal injury). With regard to the vehicle damage, referring to the Wisconsin Motor Vehicle Report Form (MV 4000) *(47)*, 4,640 or 33.9 percent were none (no damage) or minor (cosmetic damage); 6,250 or 45.7 percent were moderate (broken or missing parts); and 2,793 or 20.4 percent were severe (salvageable) or very severe (total loss).

Factors contributing to crashes in the database were categorized into four groups: driver characteristics, highway and traffic factors, environmental factors and crash characteristics. Driver characteristics include driver's age, gender, usage of safety restraints and whether the driver was driving under the influence of alcohol or drugs. Highway and traffic factors include the highway geometric characteristics, highway class and traffic control types. Environmental factors include weather, light and roadway surface conditions. Crash characteristics include the manner of collision which describes the orientation that vehicles collided, and the collision type which indicates the types of vehicles that collided with each other. The detailed description of selected variables is shown in Table 1.

**TABLE 1 Description of Selected Variables**

| Category | Variable | Type and Value | Description | Frequency | Percentage |
|---|---|---|---|---|---|
| **Driver Characteristics** | AGE | **Categorical** | Driver age | | |
| | | 1 | Young (<25) | 3,805 | 27.8% |
| | | 2 | Middle (25-55) | 7,688 | 56.2% |
| | | 3 | Old (>55) | 2,190 | 16.0% |
| | GENDER | **Dummy** | Male driver | 7,047 | 51.5% |
| | DUI | **Dummy** | Driver under the influence of drugs or alcohol | 524 | 3.8% |
| | SAFETY | **Dummy** | Safety restraints | 13,323 | 97.4% |
| **Highway and Traffic Factors** | ROADHOR | **Dummy** | Horizontal curve | 1,045 | 7.6% |
| | ROADVERT | **Dummy** | Vertical curve | 1,826 | 13.3% |
| | HWYCLASS | **Categorical** | Highway class | | |
| | | 1 | Urban city highway | 9,909 | 72.4% |
| | | 2 | Urban state highway | 3,549 | 25.9% |
| | | 3 | Urban interstate highway | 225 | 1.7% |
| | TRFCONT | **Categorical** | Traffic control | | |
| | | 1 | Four-way stop sign (intersection) | 344 | 2.5% |
| | | 2 | Two-way stop sign (intersection) | 1,491 | 10.9% |
| | | 3 | Signal (intersection) | 4,478 | 32.7% |
| | | 4 | Yield or no control (intersection) | 1,988 | 14.5% |
| | | 5 | No control (segment) | 5,382 | 39.4% |
| **Environmental Factors** | WTHRCOND | **Categorical** | Weather condition | | |
| | | 1 | Clear | 7,290 | 53.3% |
| | | 2 | Cloudy | 4,118 | 30.1% |
| | | 3 | Rain | 1,289 | 9.4% |
| | | 4 | Snow/hail | 986 | 7.2% |
| | LGTCOND | **Categorical** | Light condition | | |
| | | 1 | Day | 10,059 | 73.5% |
| | | 2 | Night without street light | 941 | 6.9% |
| | | 3 | Night with street light | 2,683 | 19.6% |
| | ROADCOND | **Categorical** | Road surface condition | | |
| | | 1 | Dry | 9,206 | 67.3% |

| | | | | | |
|---|---|---|---|---|---|
| | | 2 | Wet | 2,448 | 17.9% |
| | | 3 | Snow/slush | 1,495 | 10.9% |
| | | 4 | Ice | 534 | 3.9% |
| **Crash Characteristics** | MNRCOLL | **Categorical** | Manner of collision | | |
| | | 1 | Head-on | 277 | 2.0% |
| | | 2 | Rear-end | 5,295 | 38.7% |
| | | 3 | Sideswipe (same/opposite direction) | 2,588 | 18.9% |
| | | 4 | Angle | 5,523 | 40.4% |
| | COLLTYPE | **Categorical** | Collision type | | |
| | | 1 | PC with PC | 10,148 | 74.2% |
| | | 2 | PC with truck | 3,243 | 23.7% |
| | | 3 | Truck with truck | 292 | 2.1% |
| **Crash Severities** | INJSVR | **Ordinal** | Injury severity level | | |
| | | 1 | O | 9,488 | 69.3% |
| | | 2 | C+B | 4,062 | 29.7% |
| | | 3 | A+K | 133 | 1.0% |
| | VEHDMG | **Ordinal** | Vehicle damage level | | |
| | | 1 | None or minor | 4,640 | 33.9% |
| | | 2 | Moderate | 6,250 | 45.7% |
| | | 3 | Severe or very severe | 2,793 | 20.4% |

## MODEL SPECIFICATIONS AND ASSUMPTIONS

Using a copula-based model, injury severity and vehicle damage indicators were jointly modeled to explore factors contributing to the crash outcomes. The joint model contains an injury severity component and a vehicle damage component. In the injury severity component, all four categories of explanatory variables: driver characteristics, highway and traffic factors, environmental factors, and crash characteristics were explored. On the other hand, in the vehicle damage component, driver characteristics were not considered because it was assumed that vehicle damage is affected by highway and traffic factors, environmental factors, and crash characteristics. The detailed discussion and explanation of the variable selection for injury severity and vehicle damage models can be found in a previous study conducted by Qin *et al.* *(35)*.

Six different copula structures were explored in this study: the Gaussian, FGM, Frank, Clayton, Joe and Gumbel copulas. The model development process comprised of the following three steps: 1) and independent model of injury severity and vehicle damage was estimated to serve as the starting point for the joint model estimation and also for purposes of comparison with the joint model, 2) copula models using the six different types of copulas were estimated, 3) and finally, the six copula models were compared with the independent model and with each other; Bayesian Information criterion (BIC) criterion was used to determine the best model (*42*).

## MODEL ESTIMATION RESULTS

### *Coefficient Estimates*

As noted earlier, six different copula models and an independent model were estimated in this study. The performance of the best five models is listed in Table 2. Based on the model goodness-of-fit, all six copula based models have a lower BIC value than the independent model. This indicates the correlations caused by unobserved factors between injury severity and vehicle damage do exist, and accounting for these dependencies can improve model accuracy. The BIC metric for the independent model and best fitting four copula models are presented in Table 2. Among the copula based models, the model with a Gaussian copula structure was found to provide the lowest BIC value thereby indicating that the model best fits the data.

**TABLE 2 Estimated Results and Model Performances**

| Models | Number of Estimated Parameters | BIC |
|---|---|---|
| Independent Model | 24 | 44,862.79 |
| Gaussian Copula Model | 29 | 44,037.98 |
| FGM Copula Model | 26 | 44,415.52 |
| Frank Copula Model | 29 | 44,071.91 |
| Clayton Copula Model | 26 | 44,698.22 |

Table 3 presents the coefficient estimates of Gaussian copula based model for injury severity and vehicle damage. The table also presents the results of the copula structure parameterization. In the table, a positive value of a coefficient in the model of injury severity (vehicle damage) represents a propensity to increase the injury severity (vehicle damage) and vice-versa for a negative value of a coefficient. On the other hand, a positive value in the copula structure parameterization represents a positive correlation between the common unobserved factors affecting injury severity and vehicle damage and a negative coefficient represents a negative dependency between the common unobserved factors affecting injury severity and vehicle damage.

Driver related factors play an important role in any crash severity studies. It can be seen from Table 3 that all human factors have significant influences on injury severity outcomes. It was found that young drivers are less likely to relate to severe injuries compared with others. This is possibly due to the higher physiological strength of younger drivers compared to elderly drivers *(39)*. A negative coefficient was also estimated for male drivers. Consistent with expectation, compliance with law is highly associated with the slight injury severity. It was found that the use of alcohol or drugs considerably relates to the probability of severe injury severity while using safety restraints dramatically decrease the probability of severe severity of injury.

Highway and traffic factors are of interest to highway and traffic engineers for designing and implementing cost-effective countermeasures to improve highway safety. Based on the coefficient estimates of the highway class for injury severity and vehicle damage, it can be seen that crashes occurring on the interstate highway are the most severe ones, followed by those occurring on state and city highways. This is possibly due to higher speeds associated with interstate facilities compared to other highway functional classes *(35)*. With regard to the traffic control types, four-way stop appears to be the safest traffic control strategy. Four way stop sign is less likely associated with severe injury severity compared to all other traffic controls at intersections and it is also less likely associated with severe severity of vehicle damage compared with all intersection traffic controls. This is plausible because four-way stop controlled intersections experience the smallest speed differentials between intersecting highways compared with others thereby leading to lower levels of injury severity and vehicle damage in the event of a crash *(35)*.

Environmental factors were also found to affect both injury severity and vehicle damage. It is interesting to note that adverse roadway conditions are more likely to be associated with slight injury severity and slight vehicle damage. This is possibly due to the reduction in speeds by drivers for cautionary reasons during adverse weather conditions *(7)*. One of the most interesting finding is with regard to the lighting conditions. It was found that crashes caused at night time are related with severe vehicle damage irrespective of the street lighting conditions. However, no such influence was found on injury severity. This can be supported by the study conducted by Qin *et al*. *(35)* in which the authors concluded that the structural design of the vehicle can protect occupants from sustaining injuries, but severe collisions may reduce the effectiveness of the protection.

With regard to the manner of collision, compared with the rear-end crashes, head-on crashes are significantly associated with the severe injury severity; both head-on and angle crashes are associated with severe vehicle damage. For the collision type, crashes between two passenger cars are significantly associated with severe injury severity and vehicle damage compared with those between a passenger car and a truck as well as between two trucks. This is possible due to the larger speed differentials between two passenger cars.

**TABLE 3 Gaussian Copula Model Coefficient Estimates and Copula Parameters**

| Gaussian Copula Ordered Probit-Ordered Probit Model | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Variable** | | **Injury Severity Component** | | | | **Vehicle Damage Component** | | | |
| | | **Coef.** | **SE** | **t** | **P > \|t\|** | **Coef.** | **SE** | **t** | **P > \|t\|** |
| **Driver characteristics** | | | | | | | | | |
| Age | Old | Base level | | | | NA | | | |
| | Middle | --- | | | | NA | | | |
| | Young | -0.24 | 0.03 | -9.67 | <0.01 | NA | | | |
| Gender | Male driver | -0.23 | 0.02 | -10.28 | <0.01 | NA | | | |
| DUI | Drug or alcohol | 0.31 | 0.05 | 6.03 | <0.01 | NA | | | |
| Safety | Safety restraints | -0.6 | 0.06 | -9.87 | <0.01 | NA | | | |
| **Highway and traffic factors** | | | | | | | | | |
| Curve | Horizontal curve | --- | | | | --- | | | |
| | Vertical curve | --- | | | | --- | | | |
| Highway class | Urban city highway | Base level | | | | Base level | | | |
| | Urban state highway | 0.10 | 0.03 | 3.76 | <0.01 | 0.06 | 0.02 | 2.68 | 0.01 |
| | Urban interstate highway | 0.18 | 0.09 | 2.15 | 0.03 | 0.35 | 0.07 | 4.78 | <0.01 |
| Traffic control | No control (segment) | Base level | | | | Base level | | | |
| | Two-way stop sign (intersection) | 0.13 | 0.04 | 3.50 | <0.01 | --- | | | |
| | Signal (intersection) | 0.13 | 0.03 | 5.09 | <0.01 | --- | | | |
| | Yield or no control (intersection) | 0.08 | 0.03 | 2.45 | 0.01 | --- | | | |
| | Four-way stop sign (intersection) | --- | | | | -0.32 | 0.06 | -5.04 | <0.01 |
| **Environmental factors** | | | | | | | | | |
| Weather condition | Clear | Base level | | | | Base level | | | |
| | Cloudy | --- | | | | --- | | | |
| | Rain | --- | | | | --- | | | |
| | Snow/hail | --- | | | | --- | | | |
| Light condition | Day | Base level | | | | Base level | | | |
| | Night without street light | --- | | | | 0.09 | 0.04 | 2.40 | 0.02 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Night with street light | --- | | | | 0.10 | 0.02 | 4.19 | <0.01 |
| Roadway condition | Dry | Base level | | | | Base level | | | |
| | Wet | --- | | | | --- | | | |
| | Snow/slush | -0.17 | 0.04 | -4.66 | <0.01 | -0.13 | 0.03 | -4.26 | <0.01 |
| | Ice | -0.17 | 0.06 | -2.88 | <0.01 | -0.10 | 0.05 | -1.93 | 0.05 |
| **Crash characteristics** | | | | | | | | | |
| Manner of collision | Rear-end | Base level | | | | Base level | | | |
| | Head-on | 0.44 | 0.07 | 5.94 | <0.01 | 0.97 | 0.07 | 14.52 | <0.01 |
| | Sideswipe (same/opposite direction) | -0.60 | 0.03 | -18.44 | <0.01 | --- | | | |
| | Angle | --- | | | | 0.64 | 0.02 | 31.58 | <0.01 |
| Collision type | Truck with truck | Base level | | | | Base level | | | |
| | Passenger car with truck | --- | | | | --- | | | |
| | Passenger car with passenger car | 0.08 | 0.03 | 2.97 | <0.01 | 0.06 | 0.02 | 2.69 | 0.01 |
| Threshold | μ1 | -0.19 | | | | -0.10 | | | |
| | μ2 | 1.74 | | | | 1.22 | | | |
| **Copula Parameters** | | **Coef.** | **SE** | **t** | **P > \|t\|** | | | | |
| Constant | | 0.11 | 0.03 | 4.12 | <0.01 | | | | |
| Passenger car with passenger car | | -0.11 | 0.02 | -4.40 | <0.01 | | | | |
| Head-on | | 0.46 | 0.07 | 6.34 | <0.01 | | | | |
| Angle | | 0.45 | 0.02 | 18.06 | <0.01 | | | | |
| Sideswipe (same/opposite direction) | | 0.39 | 0.03 | 11.37 | <0.01 | | | | |

Notes: “NA” represents “not applicable”; “---” represents the variable is not statistically at 5% level of significance.

The estimated copula parameters offered additional insight about the dependencies between injury severity and vehicle damage. In determining variables for the copula structure, we first select all candidate variables, and then remove variables that are not statistically significant. In Table 3, only the parameters for the copula structure that have been considered to be statistically significant at 5 percent level of significance are included.

The results highlight the existence of dependencies between injury severity and vehicle damage caused by the common unobserved factors. A positive parameter indicates that the dependencies between injury severity and vehicle damage caused by the common unobserved factors for the specific type of crashes are positive, and a negative parameter indicates that the dependencies between injury severity and vehicle damage caused by the common unobserved factors for the specific type of crashes are negative. It is interesting to note that the dependencies vary with different characteristics of crashes including manners of collision and collision types. With regard to three manners of collision: head-on, angle and sideswipe, the dependencies between injury severity and vehicle damage caused by the common unobserved factors were found to be positive. The magnitude of copula parameters implies that the highest level of dependency between injury severity and vehicle damage is for head-on crashes, followed by angle and sideswipe crashes. Also, the dependencies between injury severity and vehicle damage for crashes between two passenger cars were shown to be negative.

***Elasticity Effects***

In the copula based model, the estimated parameters alone are not sufficient to describe the magnitude of the effect of an independent variable on the probability of each vehicle damage or injury severity category. Therefore, the elasticity effects for all independent variables with regard to both injury severity and vehicle damage were calculated and are presented in Table 4. The detailed discussion on the methodology for calculating elasticity effects in a copula based model can be found in Eluru and Bhat *(48)*.

In general, the effects of independent variables on injury severity and vehicle damage shown in Table 4 are consistent with those described in Table 3. More specifically, the presence of young and male drivers decreases the probability of severe injury severity, the use of drug or alcohol significantly increase the probability of severe injuries, and using safety restraints dramatically decreases the probability of severe injuries especially the type A or fatal injuries. With regard to highway and traffic factors, roadways with higher speed limit increase the probability of both severe injuries and vehicle damage levels. Four-way stop controlled intersections decrease the probability of severe crash outcomes. In terms of the environmental factors, adverse roadway surface conditions seem to decrease the probability of injury type B or C and type A and K, as well as decreasing the probability of moderate and severe vehicle damages. Night time with or without street lights increases the probability of severe vehicle damages, but the effects of it on injury severity were not statistically significant. The crash characteristics describe the manner and vehicle type of a collision. Head-on crashes have the most significant impacts on increasing severe crash severities, and collisions between two passenger cars are the most severe ones among all collision types.

## TABLE 4 Elasticity Effects for Vehicle Damage and Injury Severity

| Variable | | Injury Severity | | | Vehicle Damage | | |
|---|---|---|---|---|---|---|---|
| | | PDO | C+B | A+K | None+ Minor | Moderate | Severe+ Very Severe |
| **Driver characteristics** | | | | | | | |
| Age | Old | | Base level | | NA | NA | NA |
| | Middle | --- | --- | --- | NA | NA | NA |
| | Young | 8.69 | -12.81 | -21.07 | NA | NA | NA |
| Gender | Male driver | 8.39 | -12.19 | -20.98 | NA | NA | NA |
| DUI | Drug or alcohol | -11.74 | 16.26 | 32.16 | NA | NA | NA |
| Safety | Safety restraints | 22.67 | -29.65 | -68.30 | NA | NA | NA |
| **Highway and traffic factors** | | | | | | | |
| Curve | Horizontal curve | --- | --- | --- | --- | --- | --- |
| | Vertical curve | --- | --- | --- | --- | --- | --- |
| Highway class | Urban city highway | | Base level | | | Base level | |
| | Urban state highway | -3.50 | 5.05 | 8.88 | -3.52 | 0.51 | 4.12 |
| | Urban interstate highway | -6.84 | 9.65 | 18.11 | -20.01 | 0.70 | 25.63 |
| Traffic control | No control (segment) | | Base level | | | Base level | |
| | Two-way stop sign (intersection) | -4.75 | 6.79 | 12.25 | --- | --- | --- |
| | Signal (intersection) | -4.94 | 7.12 | 12.51 | --- | --- | --- |
| | Yield or no control (intersection) | -3.05 | 4.39 | 7.78 | --- | --- | --- |
| | Four-way stop sign (intersection) | --- | --- | --- | 19.23 | -4.78 | -20.44 |
| **Environmental factors** | | | | | | | |
| Weather condition | Clear | | Base level | | | Base level | |
| | Cloudy | --- | --- | --- | --- | --- | --- |
| | Rain | --- | --- | --- | --- | --- | --- |
| | Snow/hail | --- | --- | --- | --- | --- | --- |
| Light condition | Day | | Base level | | | Base level | |
| | Night without street light | --- | --- | --- | -5.05 | 0.65 | 5.98 |
| | Night with street light | --- | --- | --- | -5.89 | 0.79 | 6.95 |
| Roadway condition | Dry | | Base level | | | Base level | |
| | Wet | --- | --- | --- | --- | --- | --- |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Snow/slush | 6.07 | -8.97 | -14.64 | 7.93 | -1.49 | -8.91 |
| | Ice | 6.04 | -8.94 | -14.47 | 5.80 | -1.07 | -6.55 |
| **Crash characteristics** | | | | | | | |
| Manner of collision | Rear-end | | Base level | | | Base level | |
| | Head-on | -16.54 | 22.34 | 47.36 | -49.29 | -9.44 | 74.56 |
| | Sideswipe (same/opposite direction) | 20.62 | -31.51 | -46.00 | --- | --- | --- |
| | Angle | --- | --- | --- | -37.86 | 4.45 | 45.30 |
| Collision type | Truck with truck | | Base level | | | Base level | |
| | Passenger car with truck | --- | --- | --- | --- | --- | --- |
| | Passenger car with passenger car | -2.84 | 4.15 | 7.03 | -3.44 | 0.56 | 3.96 |

Notes: “NA” represents “not applicable”; “---” represents the variable is not statistically significant at 5% level of significance.

## SUMMARY AND CONCLUSIONS

Traffic safety is an important issue with serious social and financial implications including injuries, fatalities and economic losses. Reducing the number of crashes and their consequences (especially the severe ones) is an important priority for transportation safety professionals. To this end, it is necessary to explore the potential causes of crash severity, so that effective countermeasures can be implemented to alleviate the crash risk.

Crash severity including injury severity and vehicle damage has been widely studied in the literature. Numerous statistical methodologies have been implemented to identify the relationships between different explanatory variables and crash severity. Irrespective of the different model assumptions and structures, failing to capture the dependencies between injury severity and vehicle damage caused by common observed and unobserved factors may lead to the biased coefficient estimates. To address this issue, a copula based ordered probit-ordered probit model is used in this study to jointly model injury severity and vehicle damage by accommodating their dependencies. Furthermore, a parameterized copula structure is used to investigate the varied dependencies between injury severity and vehicle damage across crashes, and the elasticity effects for all independent variables were calculated to explore their effects on the probability of each injury severity and vehicle damage category.

Six copula based models including Gaussian, FGM, Frank, Clayton, Joe and Gumbel copula models and an independent model were tested in this study. The comparison of the model estimations shows that the copula based models had a better goodness-of-fit than the independent model which indicates the existence of dependencies between injury severity and vehicle damage. Among the copula based models, the Gaussian copula model had the best model performance with the lowest BIC value.

The Gaussian copula model reveals that human factors have significant influences on injury severity. Young drivers are less likely to be associated with severe injuries than others. Males have a lower probability of suffering severe injury severity compared with females. Using alcohol or drug dramatically increases the injuries and using safety restraints considerably decreases the probability of severe injuries. The crash severity on interstate highways is increased due to the higher speed. Four-way stop controlled intersections may be safer than others as both injury severity and vehicle damage are decreased. When compared with normal roadway conditions, adverse surface decreases the crash severity due to the reduced traveling speed. Night time seems to increase the probability of severe vehicle damage but it is not statistically significant for the injury severity model. Compared with the rear-end crashes, head-on crashes increase the probability of severe injuries and both head-on and angle crashes increase the probability of severe vehicle damage. The crash severity for crashes between two passenger cars may be increased due to the larger speed differentials between two vehicles.

The estimated copula parameters offer additional insight about different patterns of dependencies between injury severity and vehicle damage across crashes. The results indicate that dependencies between injury severity and vehicle damage are positive for head-on, angle and sideswipe crashes, while the dependencies are negative for the crashes between two passenger cars. These conclusions indicate that the dependencies between injury severity and vehicle damage can vary across different crashes. In summary, this study offers a more accurate model structure of predicting crash severity, and it is anticipated that this study can shed light on help develop cost-effective countermeasures to improve traffic safety.

One limitation of the study is that it employs only two vehicle crashes for the analysis. The findings are not directly transferable to crashes involving single vehicles or more than two vehicles. These are avenues for future research. From a practice perspective, the availability of vehicle damage information for roadway crashes might also influence applicability of the proposed framework. However, it is important to recognize that while vehicle damage component of the model might not be employed, the model results obtained for severity analysis can be directly employed. The injury severity estimates obtained through our two dependent variable analysis have been "purified" by considering dependency between the two variables. Hence, the states with no vehicle damage would continue using the injury severity model independently. However, from our analysis, it is evident that considering vehicle damage – an objective indicator of crash severity – might enhance crash severity analysis (*35*). Therefore, to accurately identify the severity of a crash, compiling vehicle damage is a recommendation from our analysis.

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